Quantum Computing on Long Time Scales: Using Nuclear Spin States as a Quantum Memory

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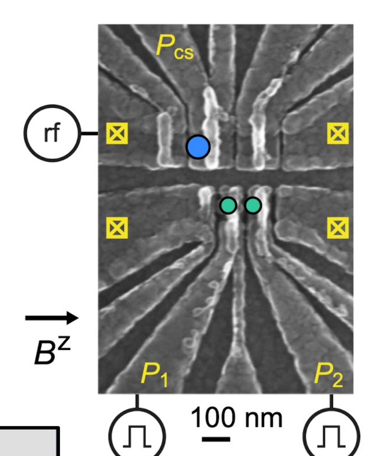
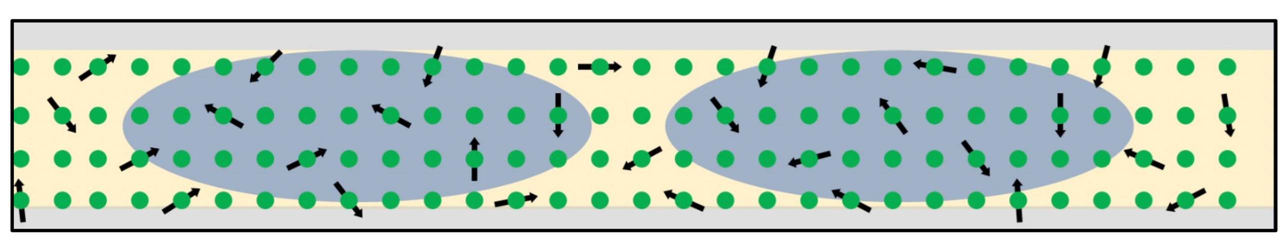
Abstract

Quantum computers are an emerging technology with many potential benefits and applications. However, decoherence poses a significant challenge to quantum computing and makes it difficult to store quantum information on long time scales. In this work, we develop a procedure that takes advantage of the longer coherence times of nuclear spin states and uses them as a quantum memory. We propose and simulate a storage-wait-retrieval process involving the transfer of information between an electron spin state and a nuclear spin state. Further, we investigate the effect of a variety of parameters on the regularity and fidelity of this procedure. The exploratory nature of this work provides valuable insight into future research aiming to develop and physically realize a quantum memory.

Physical System

To inform our simulation work, we assumed the set-up of a gate-defined double quantum dot on a Si/SiGe heterostructure (often used experimentally within the Nichol Group). Two potential wells containing two electrons are constructed using small electrical circuits. By applying different voltages to the gates, we can manipulate the electron spin state and, as an effect, alter the nuclear spin state.



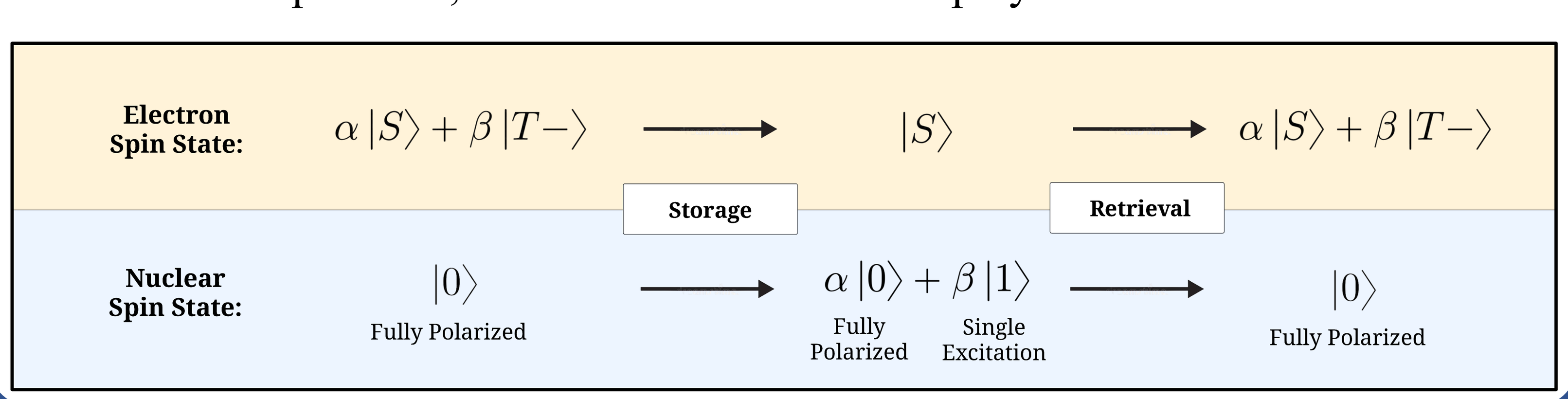


Informational States

To physically realize our quantum information, we use the spins of both the electrons and Silicon nuclei. As we are dealing with two electrons, we have four entangled electron spin states. Since we only need two states for our qubits, we use the two electron spin states with the lowest energy: the spin singlet, $|S\rangle$, and polarized triplet, $|T_{-}\rangle$. These two entangled states become our informational $|0\rangle$ and $|1\rangle$ respectively.

$$|0\rangle = |S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$
$$|1\rangle = |T-\rangle = |\downarrow\downarrow\rangle$$

As our aim is to utilize the nuclear spin state as a quantum memory, we should transfer the electron informational state to the Silicon nuclei—thus, we must represent our information with the nuclear spins as well. In preliminary work, we initialize a fully polarized nuclear spin state to realize our informational $|0\rangle$. Additionally, a single magnon excitation of this fully polarized state becomes our informational $|1\rangle$. The memory procedure involves transferring the electron state to the nuclear spin state, and vice versa. This is displayed in the schematic below:

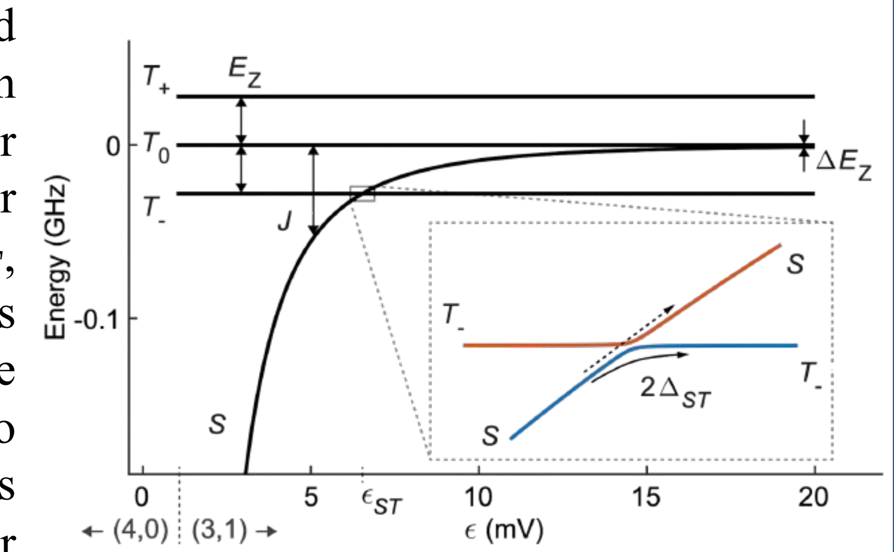


Simulation Set-up

To test and explore the memory effect and our proposed procedure, we simulate our physical system in Matlab. The Hamiltonian for this system is given by

$$H = J(\epsilon)(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2) + B \cdot \boldsymbol{I} + H_{hf}$$

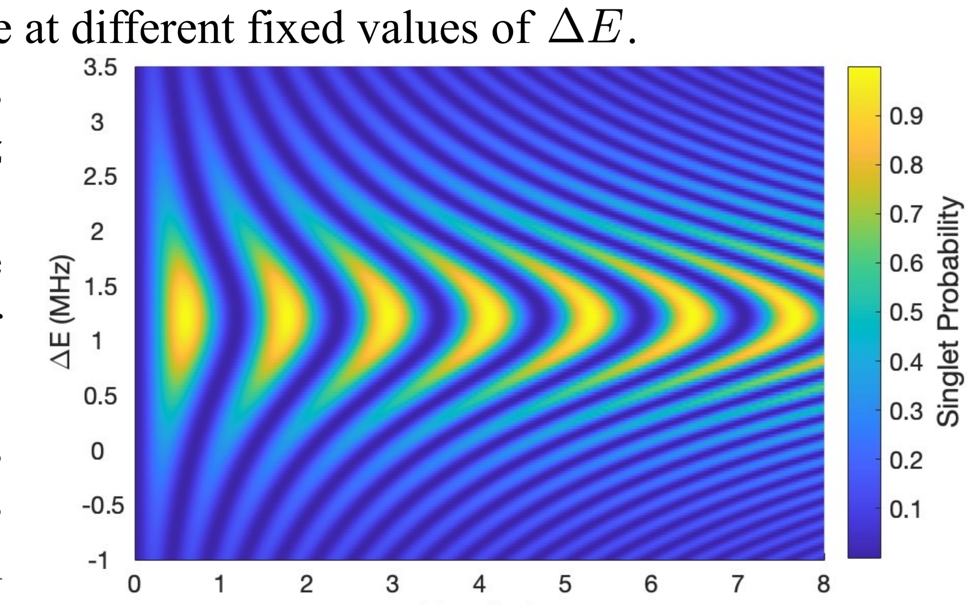
where $H_{hf} = \bar{A}\sum_{k}\sum_{l=1}^{2}\delta(\boldsymbol{R}_k - \boldsymbol{r}_l)(\boldsymbol{I}_k \cdot \boldsymbol{S}_l)$. Here, $J(\epsilon)$ is the exchange interaction, $\boldsymbol{S}$ and $\boldsymbol{I}$ are the electron and nuclear spin states respectively, and B is the external magnetic field. A parameter of particular interest for our quantum memory procedure is the detuning voltage, ϵ, as it has a large effect on the transitions between the $|S\rangle$ and $|T_{-}\rangle$ states and is something we can control with the gate voltages in our device. At a specific detuning near the unperturbed crossing point ϵ_{ST}, the energy of the $|S\rangle$ and $|T_{-}\rangle$ states become equal. At this detuning, the system can freely undergo $|S\rangle$ - $|T_{-}\rangle$ transitions, which makes this the ideal detuning value for our memory procedure.



In our initial simulations, we approximated the dynamics of the system by modeling the nuclear spins semi-classically to reduce the computational time. However, after significant analysis, we have determined that a fully quantum mechanical model of the nuclear spin states is necessary to simulate the entanglement between the spins and thus to observe the full memory oscillations.

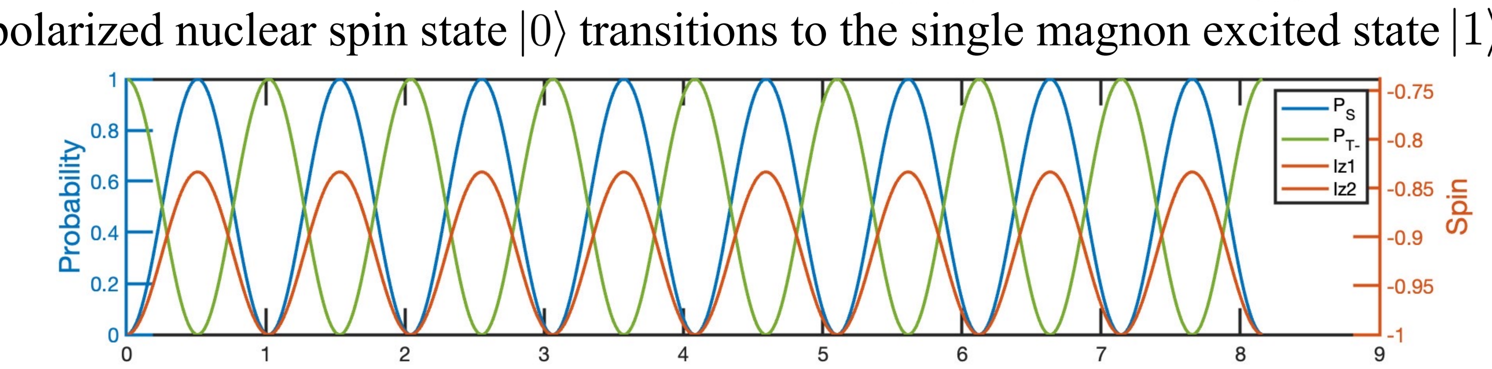
Memory Oscillations

To determine the optimal detuning value for our procedure, we simulated the $|S\rangle$ - $|T_{-}\rangle$ transitions over a wide range of ΔE, where we have defined $\Delta E \equiv E_S - E_{T-}$ as the energy difference between the $|S\rangle$ and $|T_{-}\rangle$ states in the absence of the hyperfine interaction. The Rabi chevron plot below displays how the electron state varies in time at different fixed values of ΔE.



From the plot, it becomes clear that at $\Delta E = 1.2$ MHz we observe full $|S\rangle$ - $|T_{-}\rangle$ oscillations and achieve the optimal detuning value for our memory procedure.

Additionally shown below is the electron spin state and z-component of the nuclear spin over time (fixed at $\Delta E = 1.2$ MHz). Here the electrons have been initialized in the $|T_{-}\rangle$ state and the nuclear spins start fully polarized. We can see that as $|T_{-}\rangle$ transitions to $|S\rangle$, the fully polarized nuclear spin state $|0\rangle$ transitions to the single magnon excited state $|1\rangle$.



Full Memory Procedure

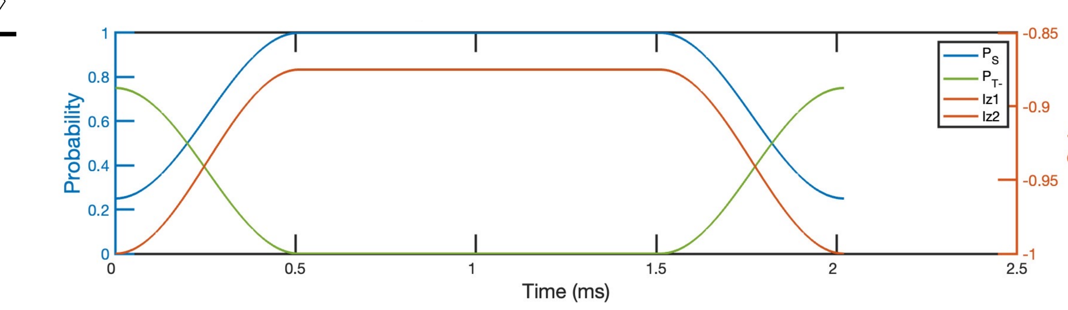
The full storage-wait-retrieval process occurs in the following five steps:

1. Initialize the nuclear spins in a polarized state
2. Prepare an arbitrary electron spin state $|\psi\rangle$ that you want to store
3. Move to $\Delta E = 1.2\,\text{MHz}$ for ½ the period of oscillations
4. Move away from the transition point to store the state (*e.g.,* $\Delta E = -100\,\text{MHz}$)
5. Return to $\Delta E = 1.2$ MHz for ½ the period of oscillations to retrieve the initial state $|\psi\rangle$.

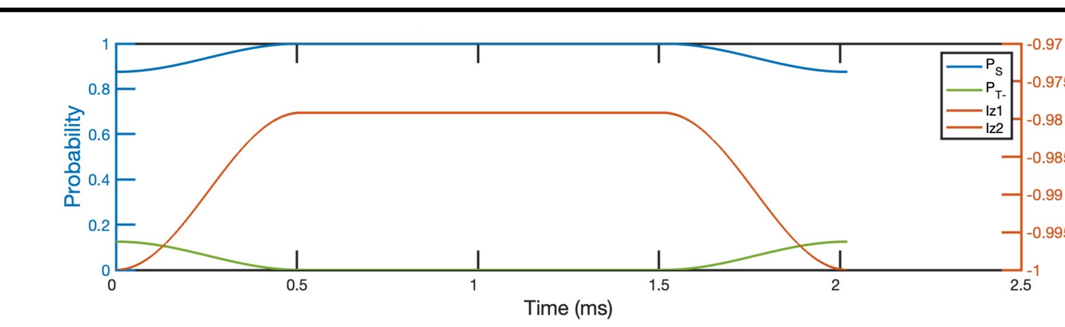
The period of oscillations, T, depends on the number of nuclear spins. Shown below, we simulate a full storage-wait-retrieval process for $N_1 = N_2 = 3$ (three nuclear spins in each of dots 1 and 2). This is significantly fewer spins than there would be interacting in an actual device, but it serves as a good proof of concept for initial testing. For three nuclear spins in each dot, we have $T = 1020\,\text{ns}$. In the trials below, we somewhat arbitrarily choose a wait time of $t_{\text{wait}} = 1000\,\text{ns}$.

Initial $|\psi\rangle = \alpha|S\rangle + \beta|T-\rangle$

$$|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix}$$



$$|\psi\rangle = \begin{pmatrix}\sqrt{\frac{7}{8}}\\ \sqrt{\frac{1}{8}}\end{pmatrix}$$



Conclusions and Future Work

In this work, we have proposed and simulated a quantum memory procedure that utilizes Silicon nuclear spin states to store information in a double quantum dot device. Due to the longer coherence times of nuclear spins relative to electron spins, there is significant potential for extensions of these simulations to help physically realize a quantum memory. Ongoing work includes introducing noise into our simulations and characterizing the resulting fidelity, exploring different initial nuclear spin states, and further developing techniques to drive random nuclear spins to the initial state of our procedure. The Nichol Group has hopes to move forward with experimental testing of this (or an adjacent) procedure in the coming years.

Acknowledgement

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