

Effective Error Channels for Magic State Distillation

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2025 Global Quantum Leap Internship

Long Update

Outline

I. Introduction

- Research question / motivation
- Magic state distillation
- Steane encoding

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II. Effective Error Channels and Results

- Process tomography
- Simulating logical circuit with effective error channels

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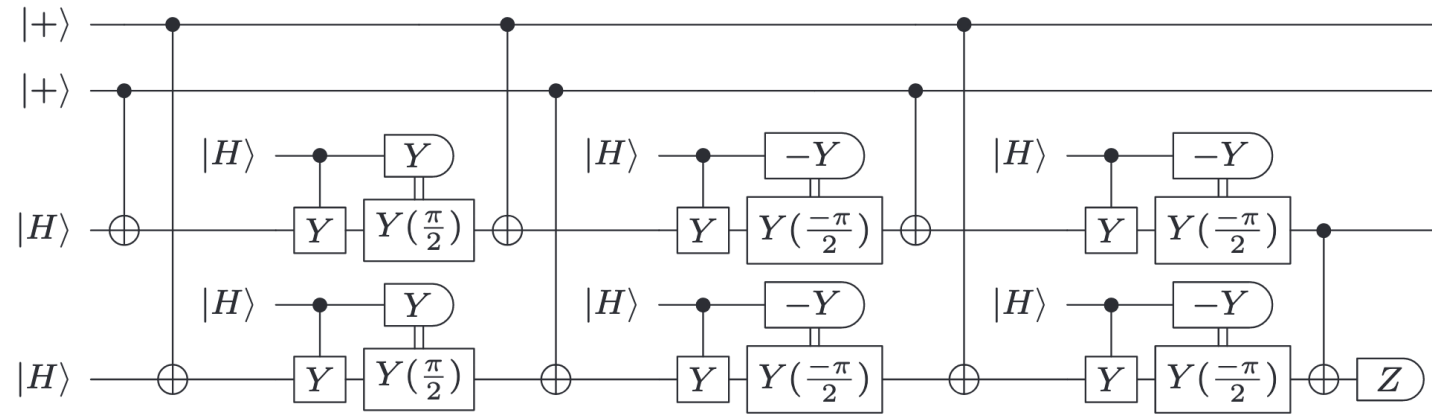
- Process tomography
- Simulating logical circuit with effective error channels

III. Outlook

- Simulating encoded circuit with base code for comparison?

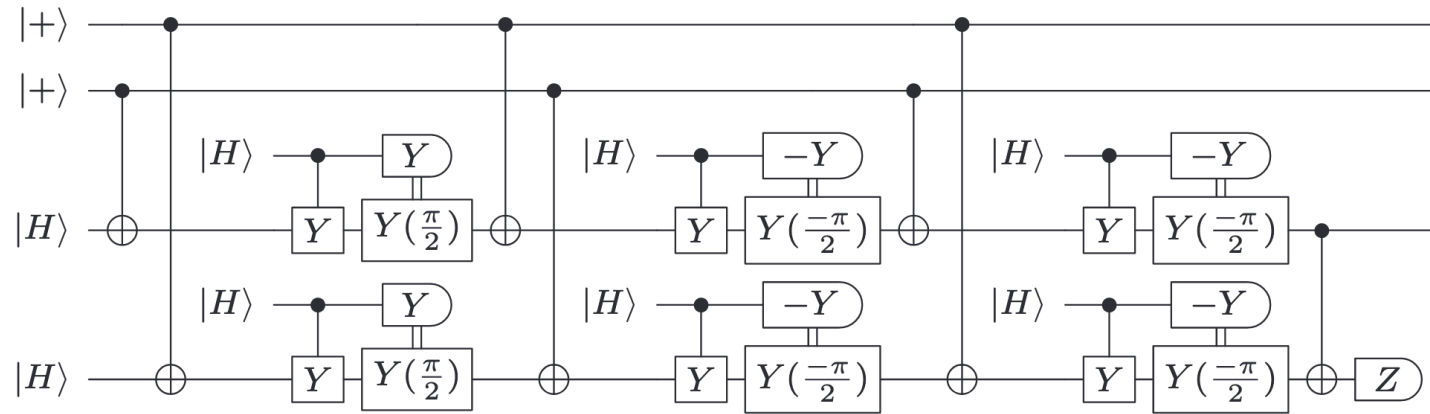
I. Introduction

Research Question

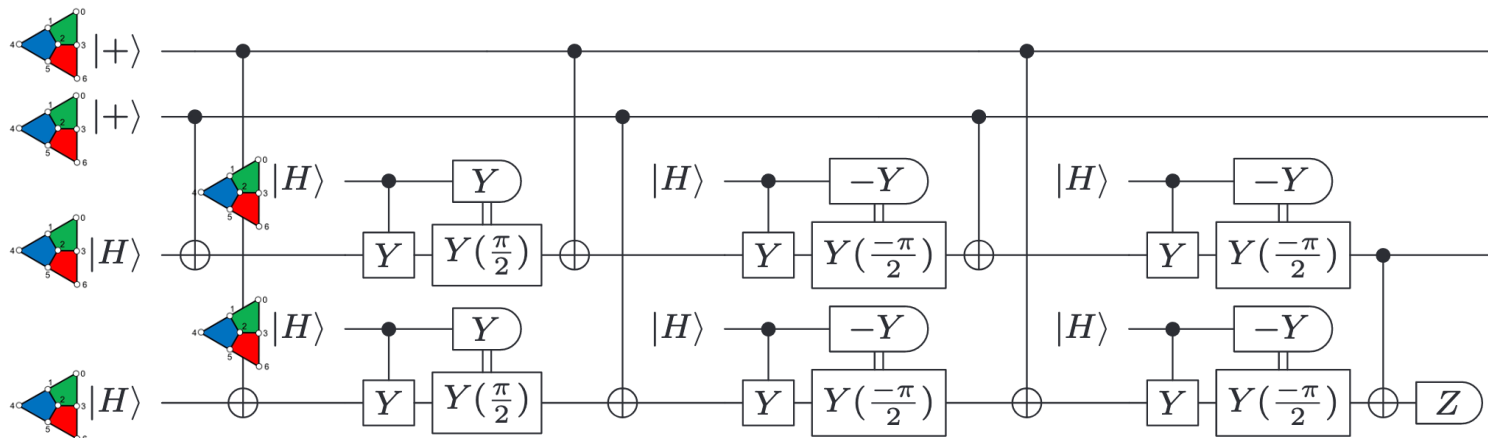


Logical Circuit (6 qubits)

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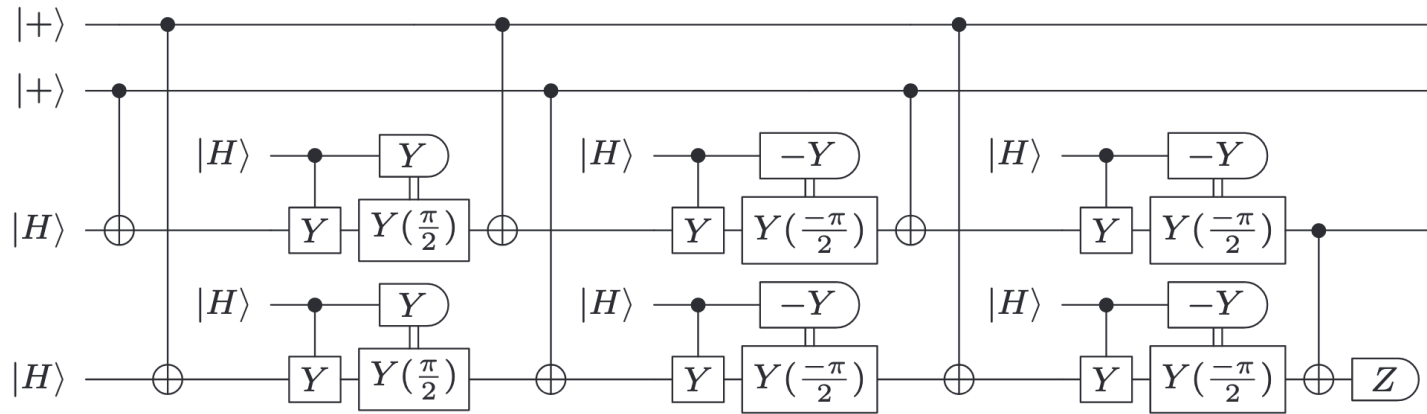


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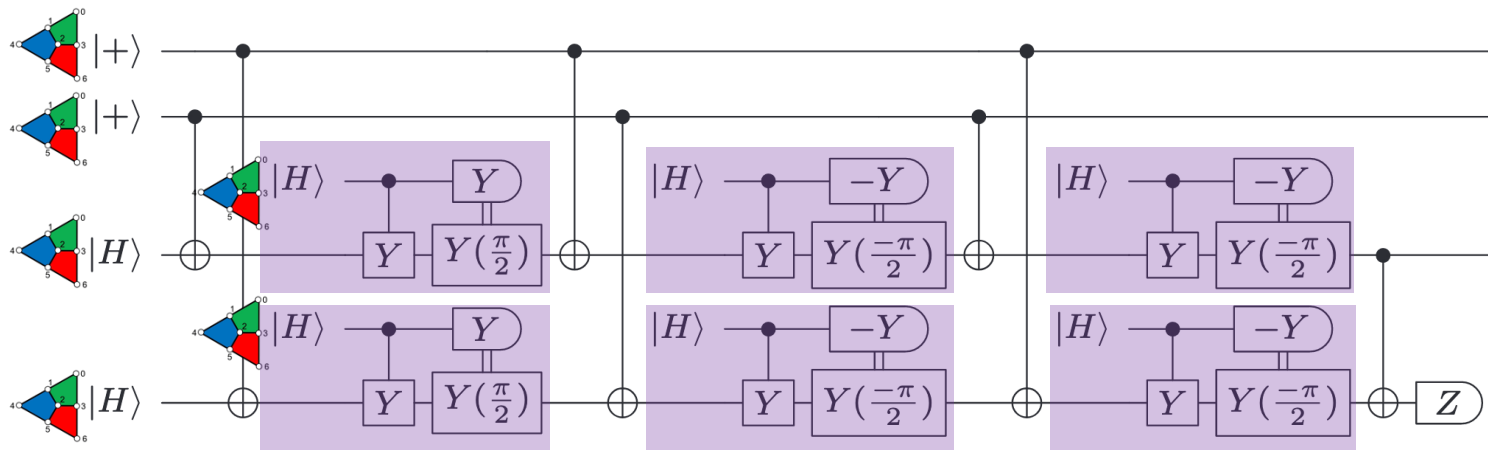


Encoded circuit with Steane code (50 qubits)

Research Question



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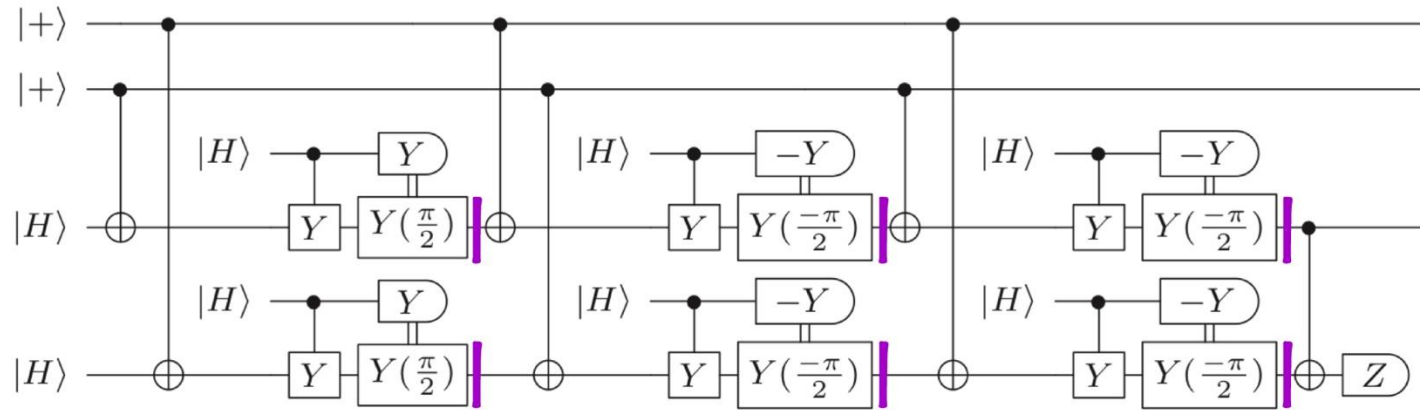


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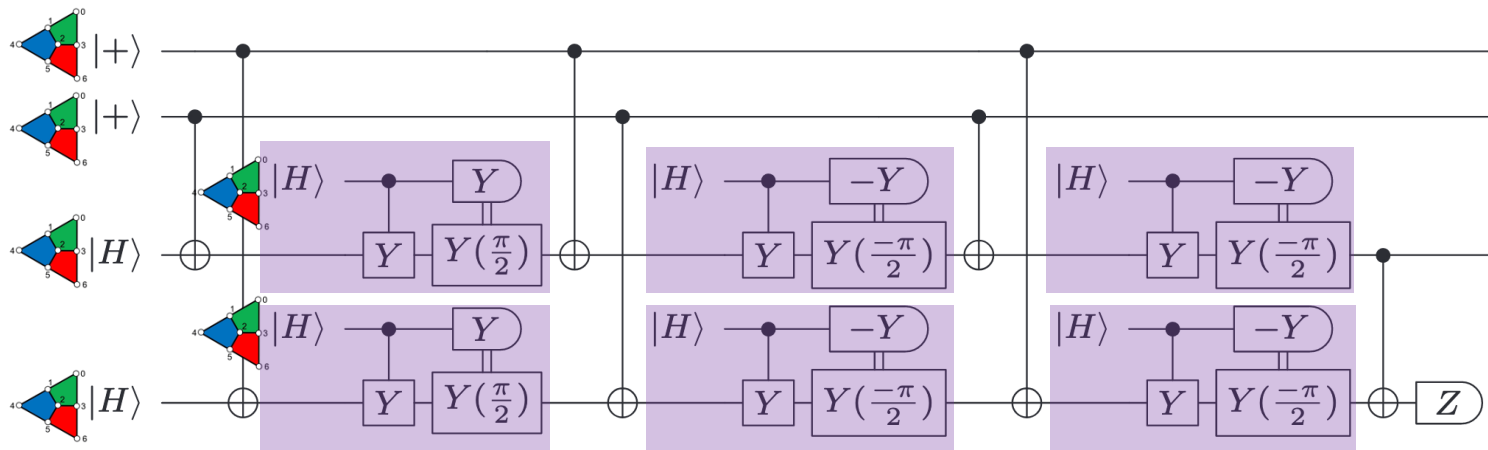
*Noise depends on
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Research Question

 = effective error channel 1



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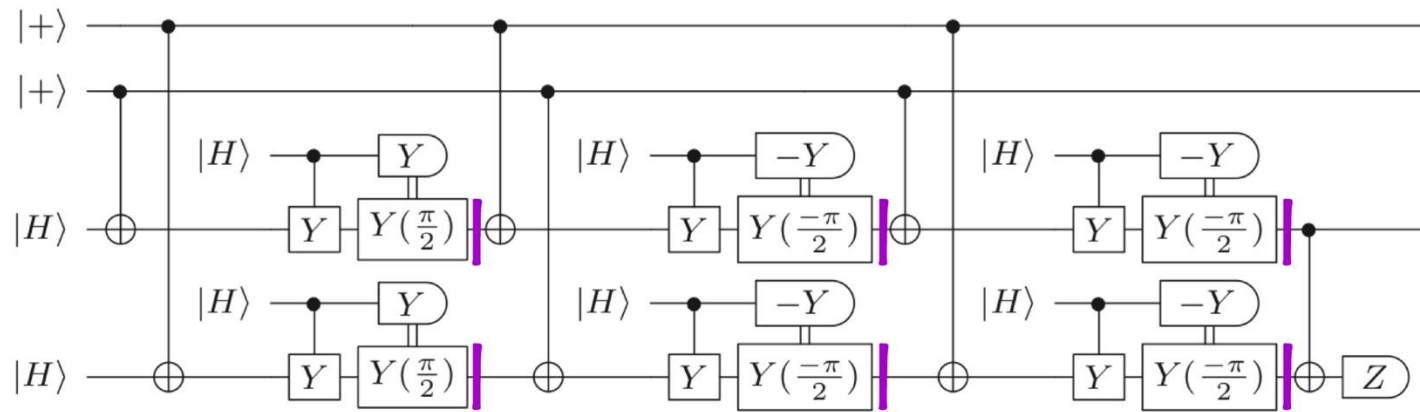


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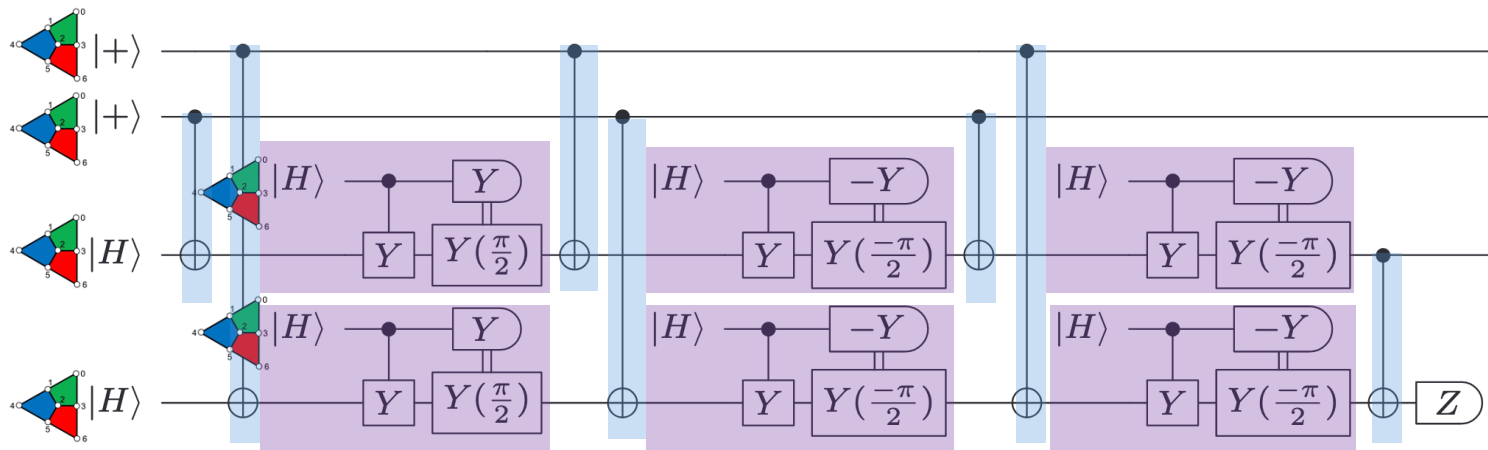
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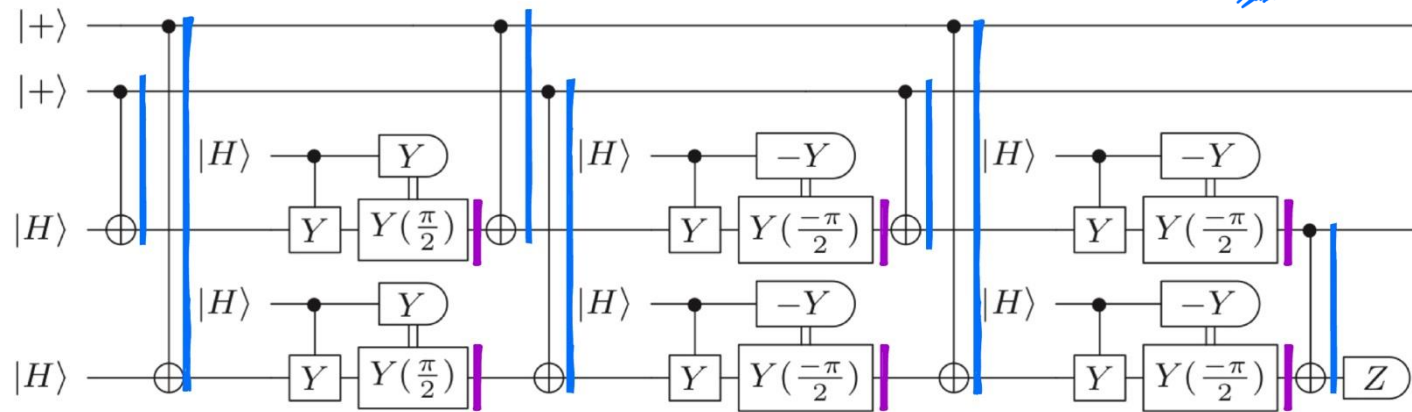
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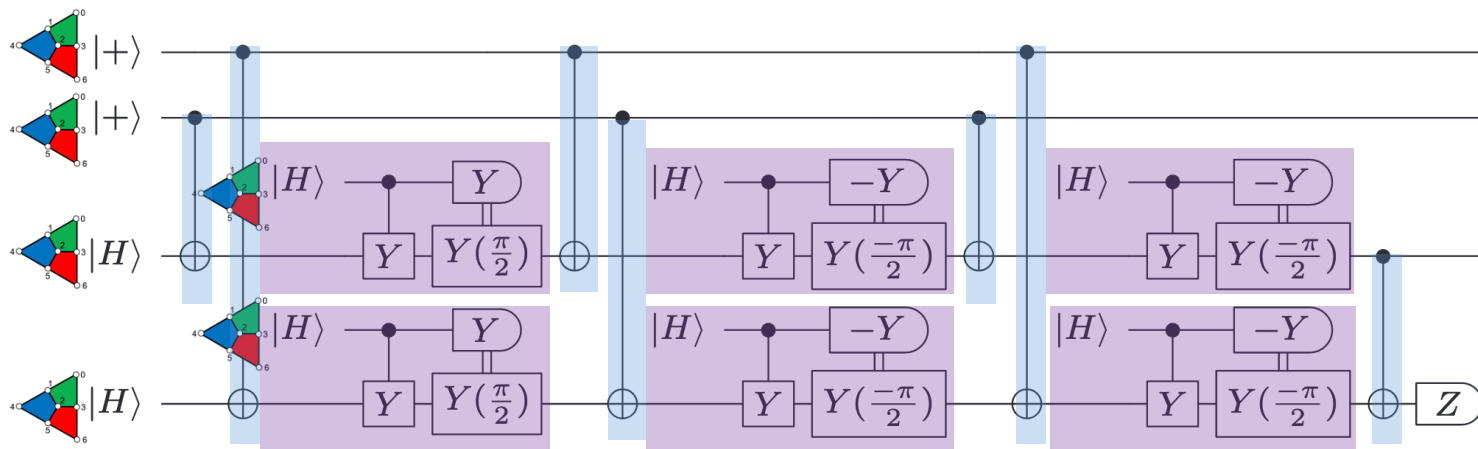
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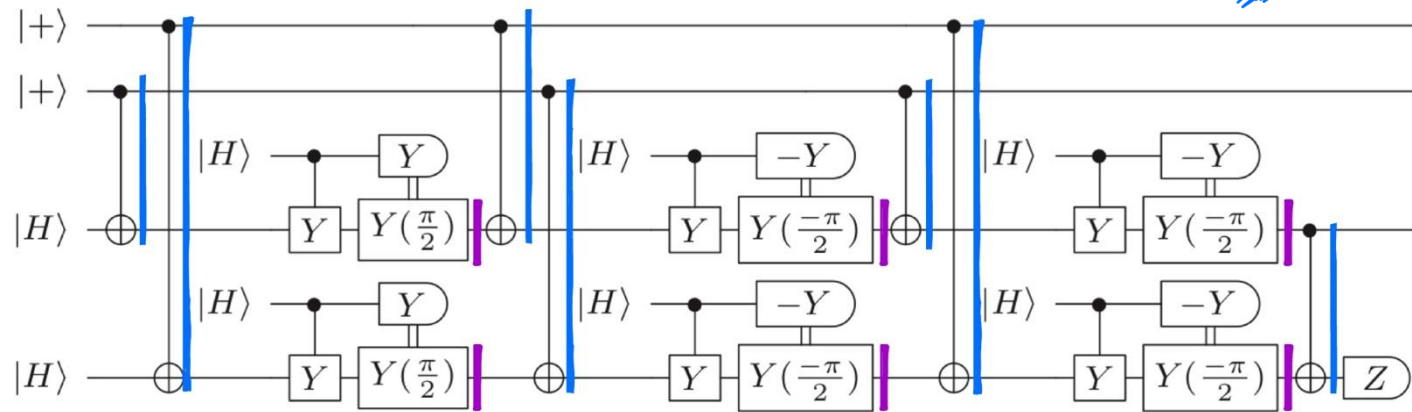


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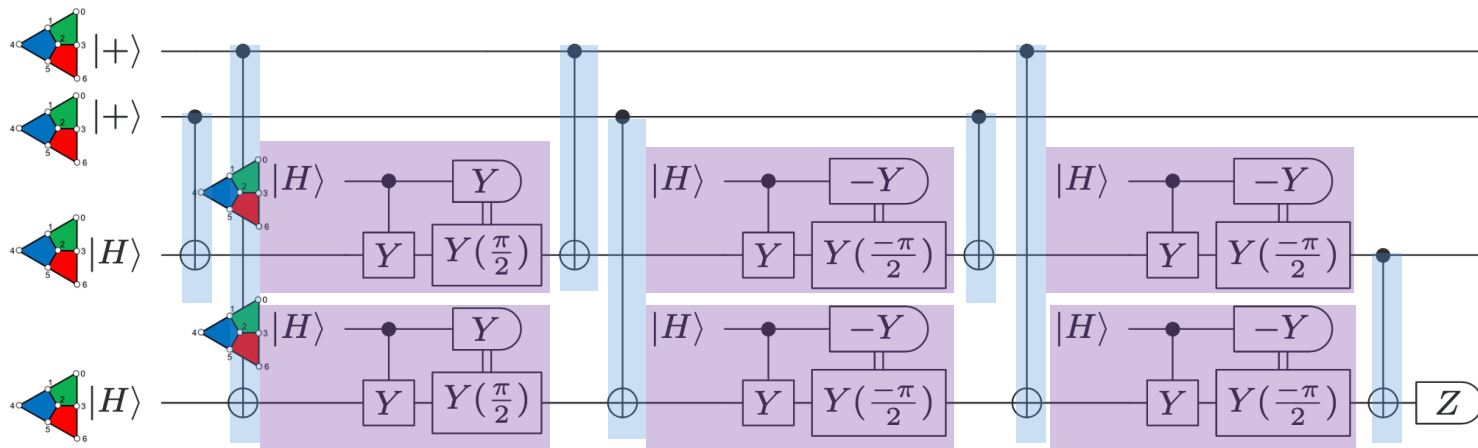
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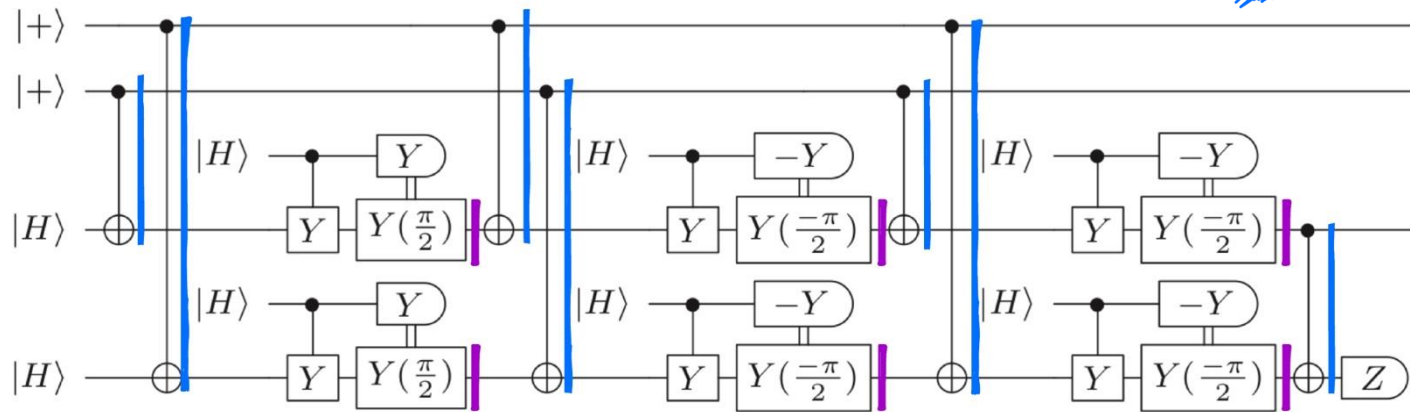
Logical circuit with effective error channels



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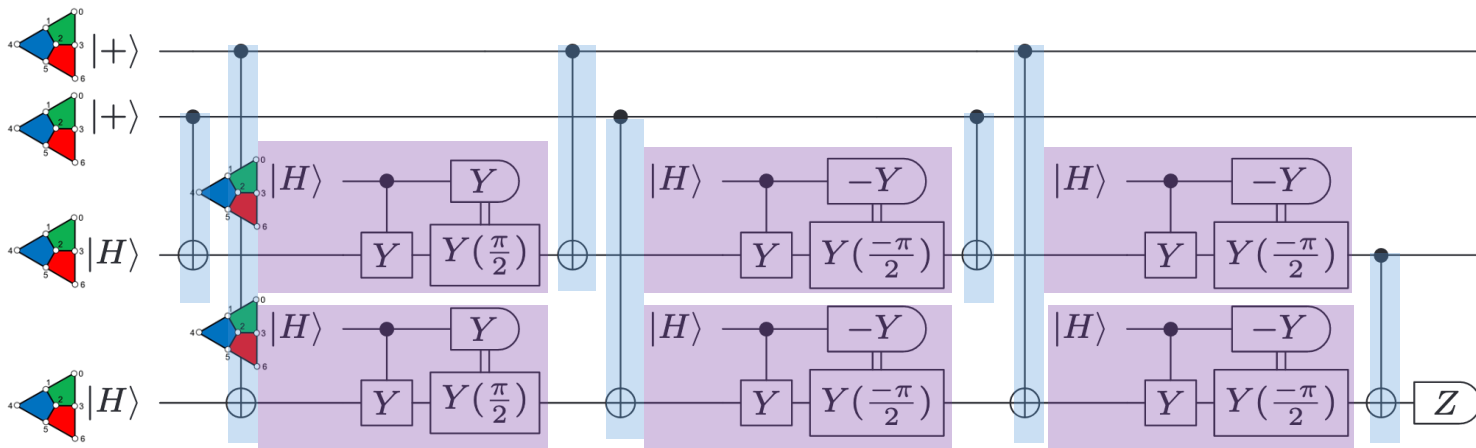
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Logical Circuit (6 qubits)

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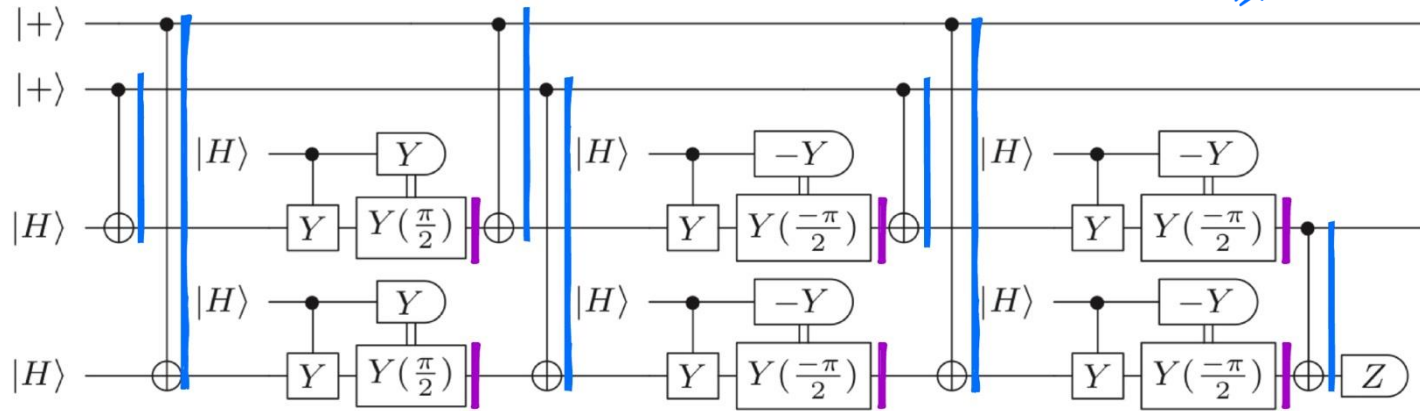


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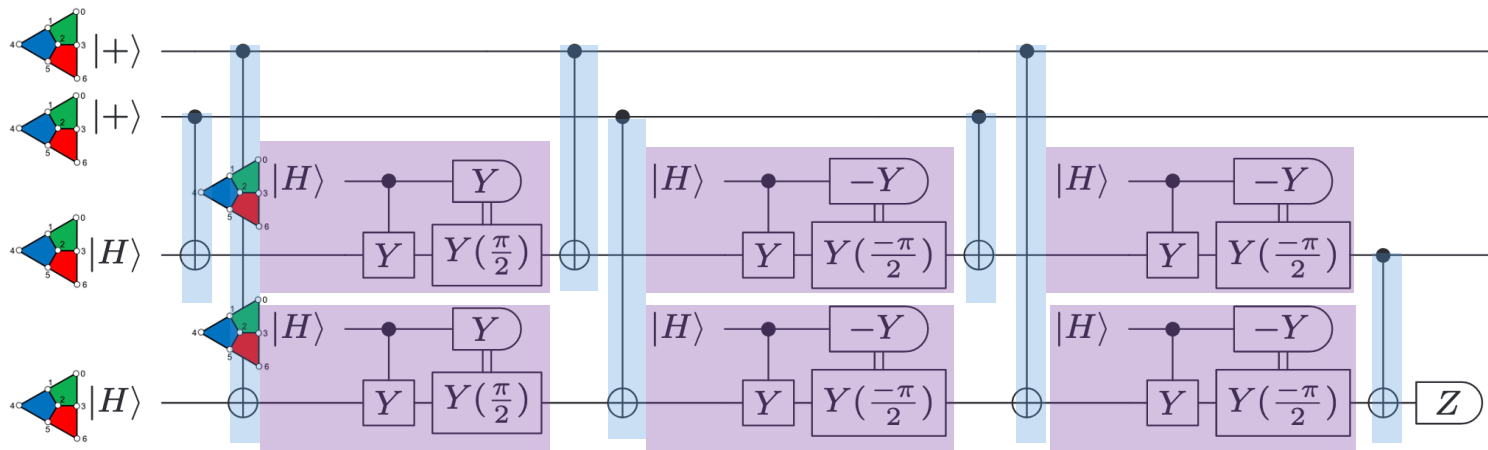
**Fully encoded circuit
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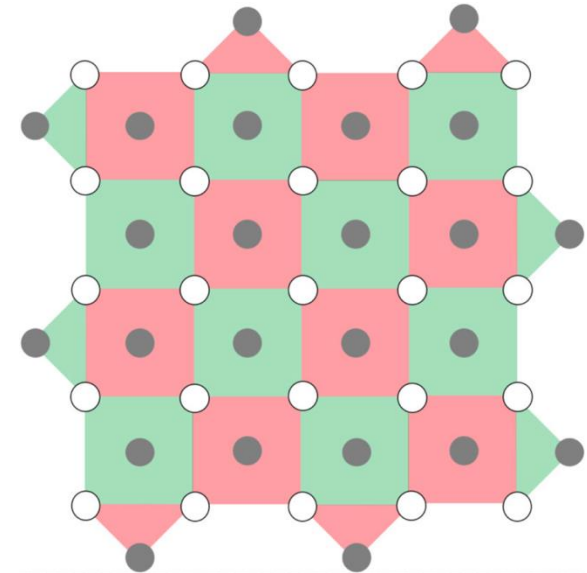
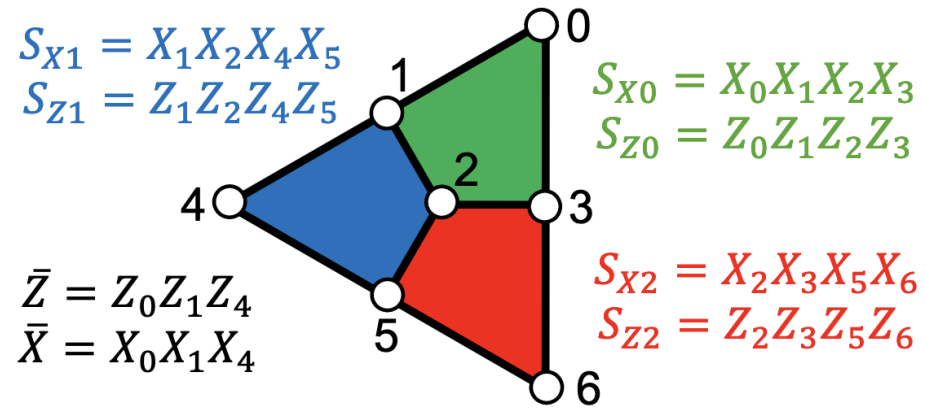
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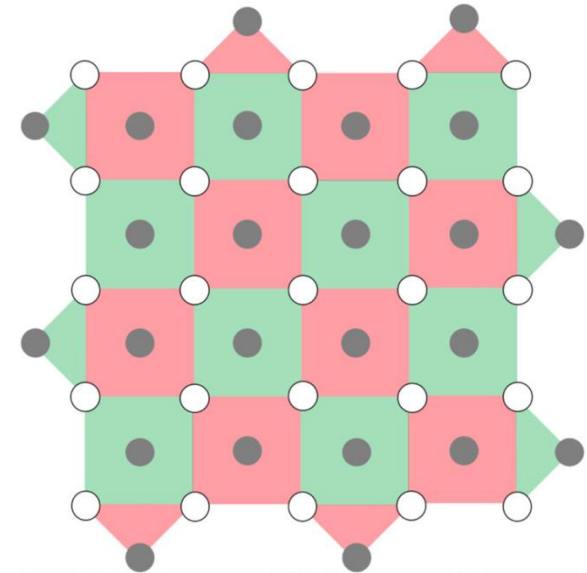
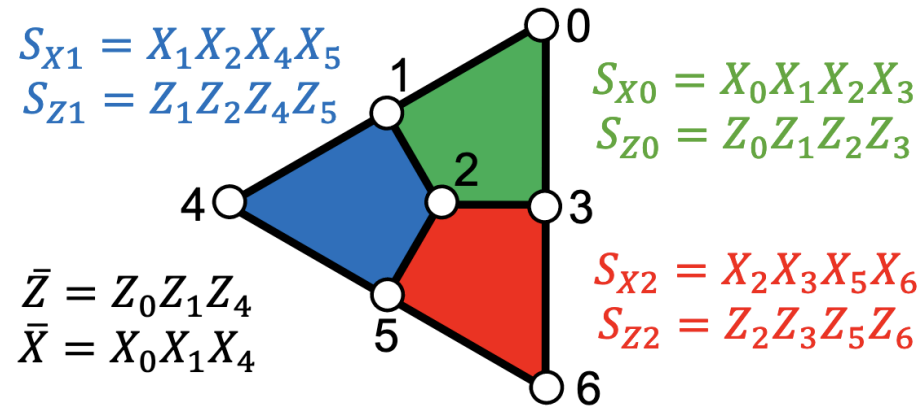
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Why QEC?



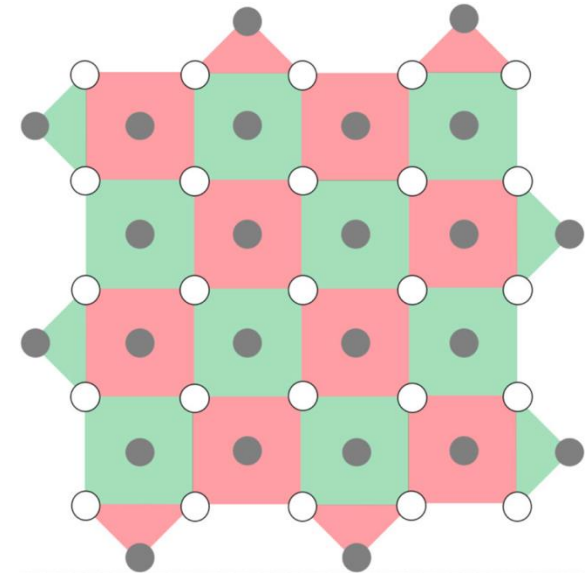
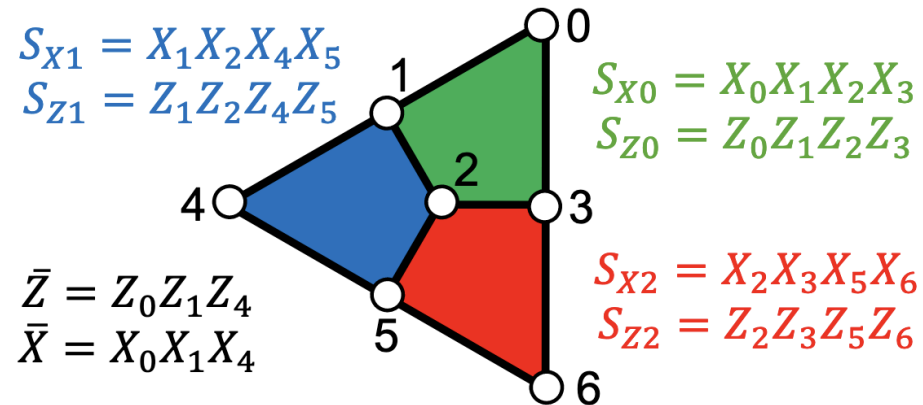
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- Physical error rates in experiment: $\sim 10^{-4}$



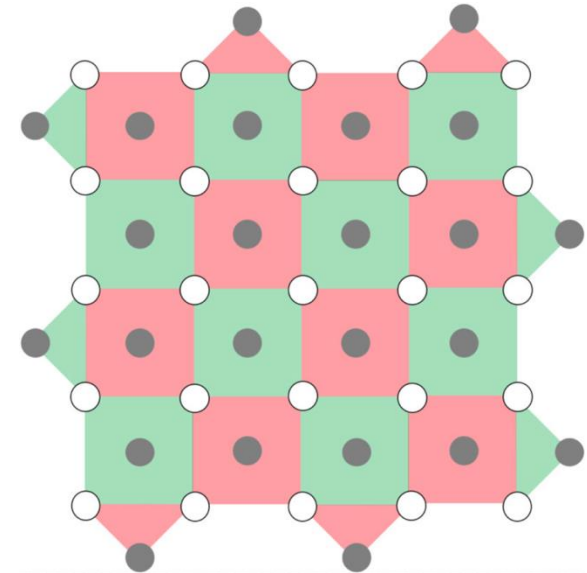
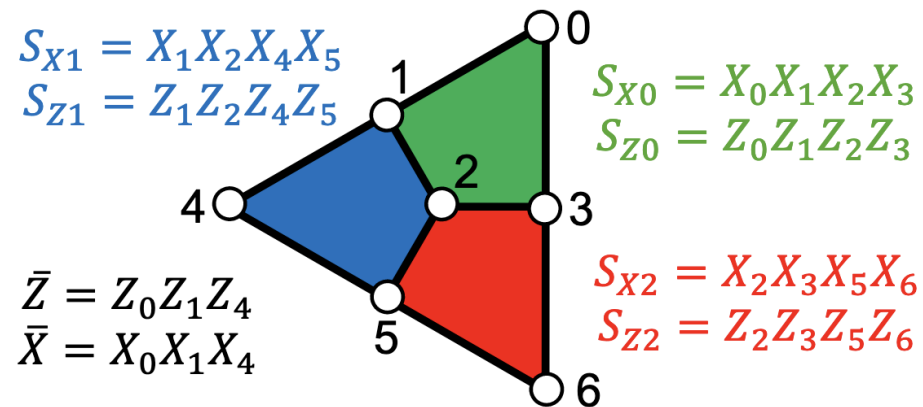
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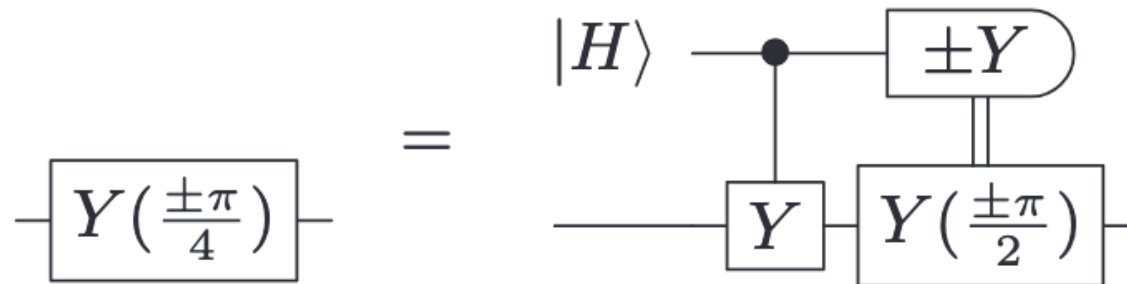
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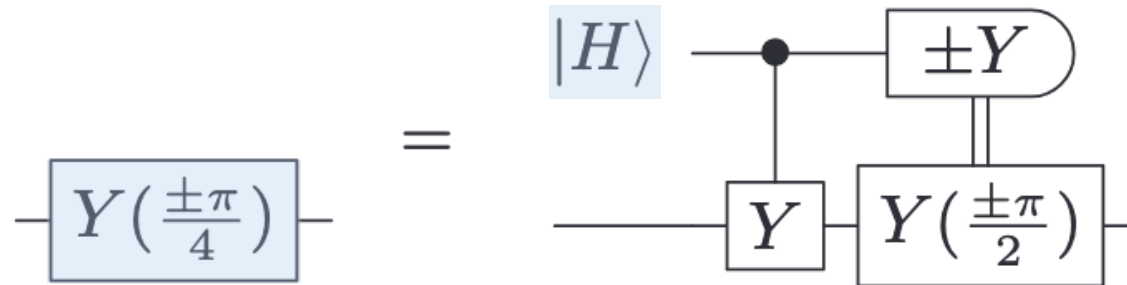


Magic state injection

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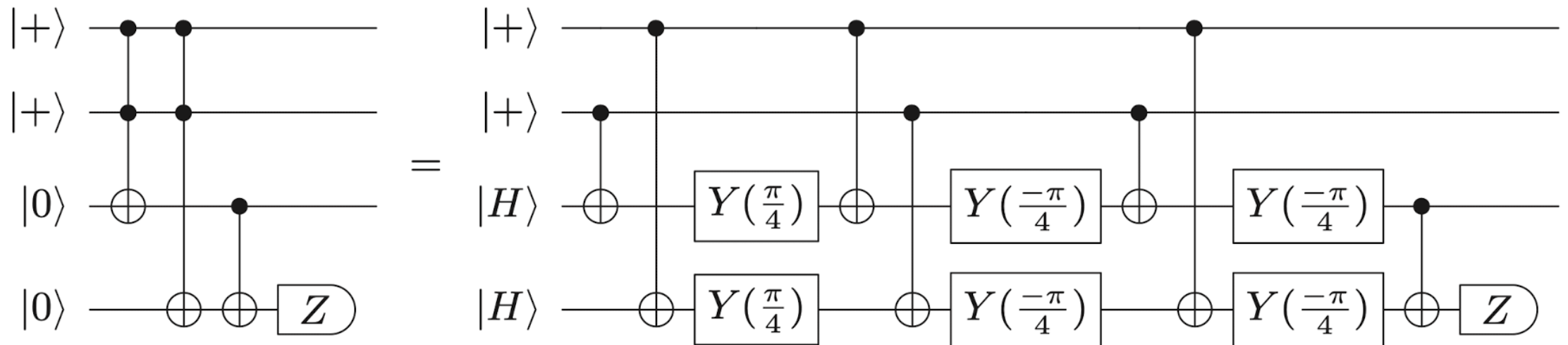


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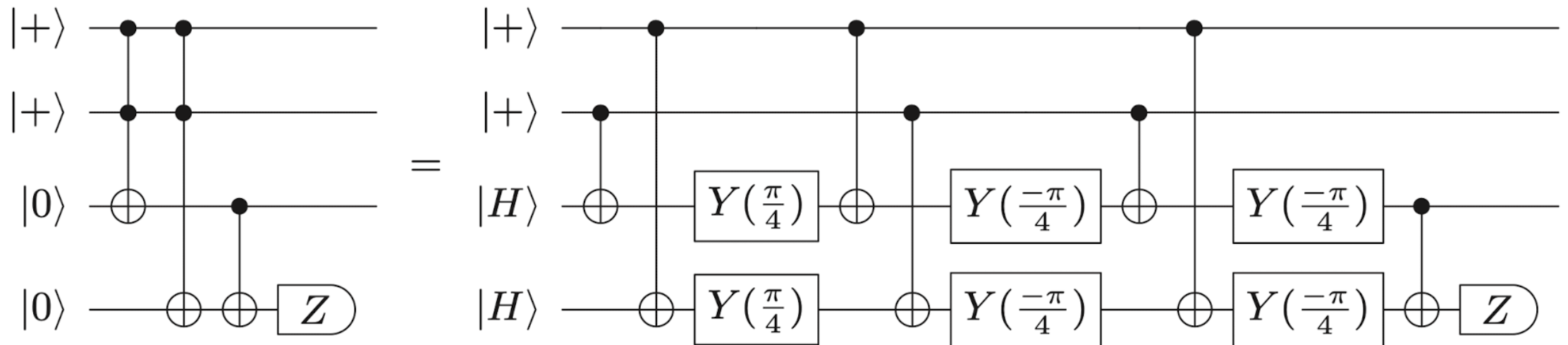


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What's going on here?

Relevant Decompositions and Identities

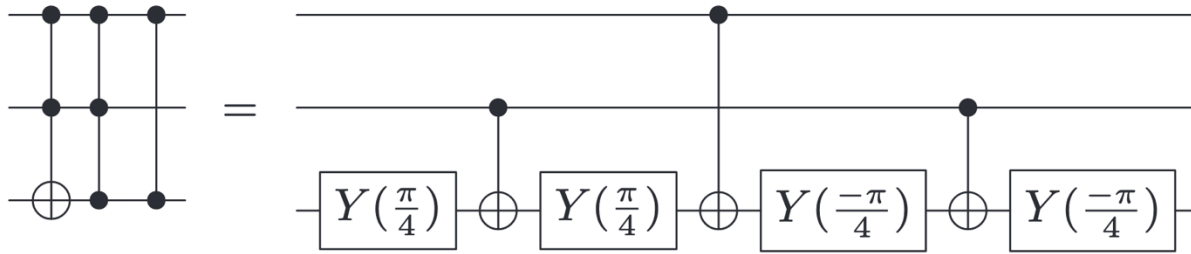


FIG. 2

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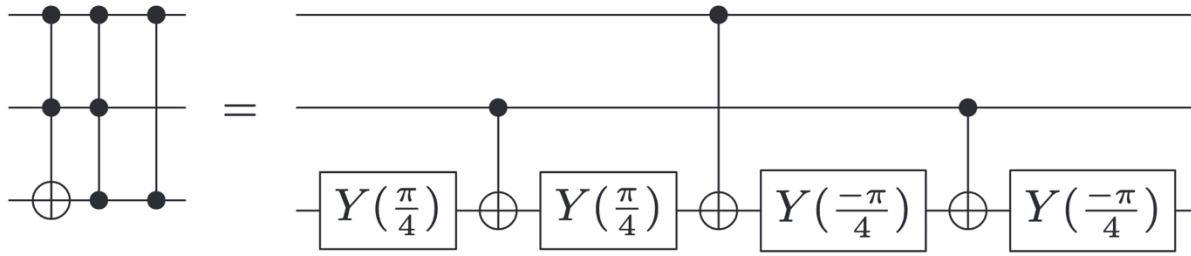


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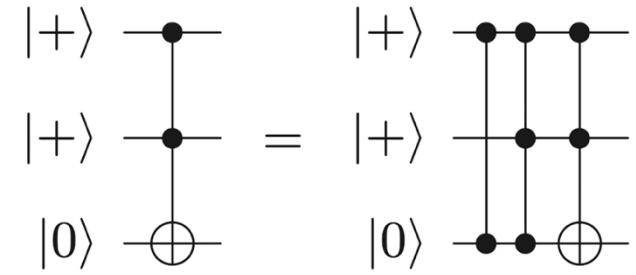


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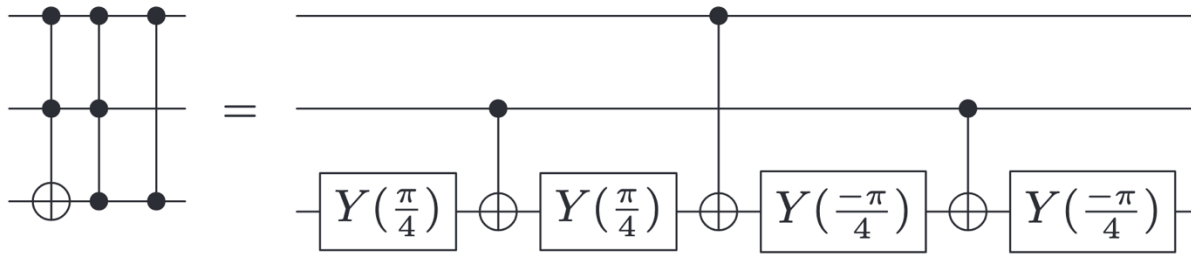


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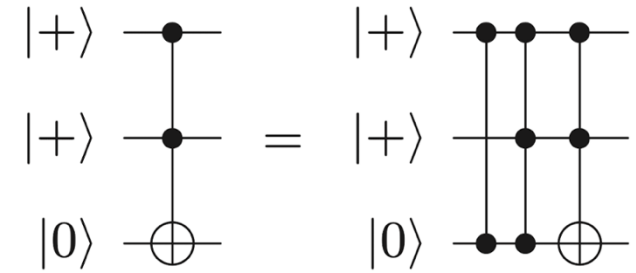


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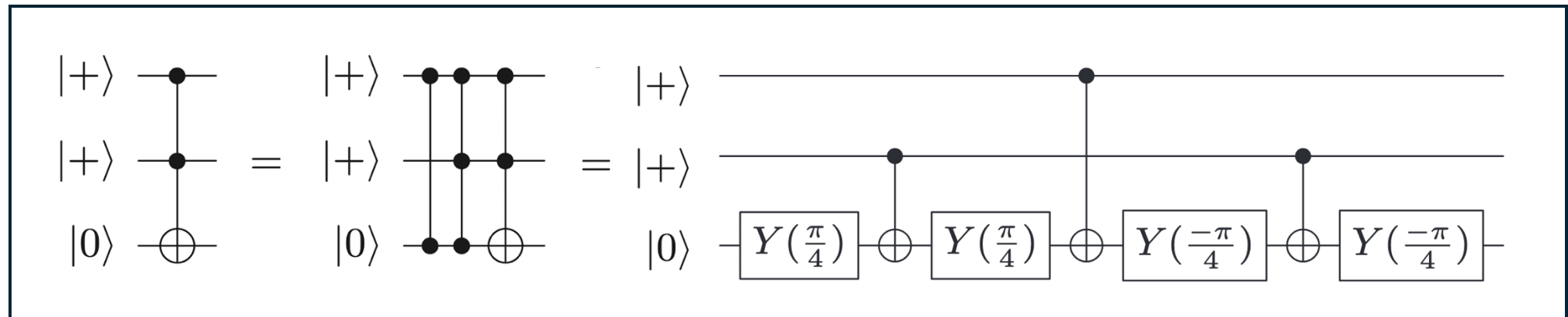


FIG. 2 + 3

$|H\rangle$ -to- $|T\text{offoli}\rangle$ Distillation Circuit (*Eastin 2013*)

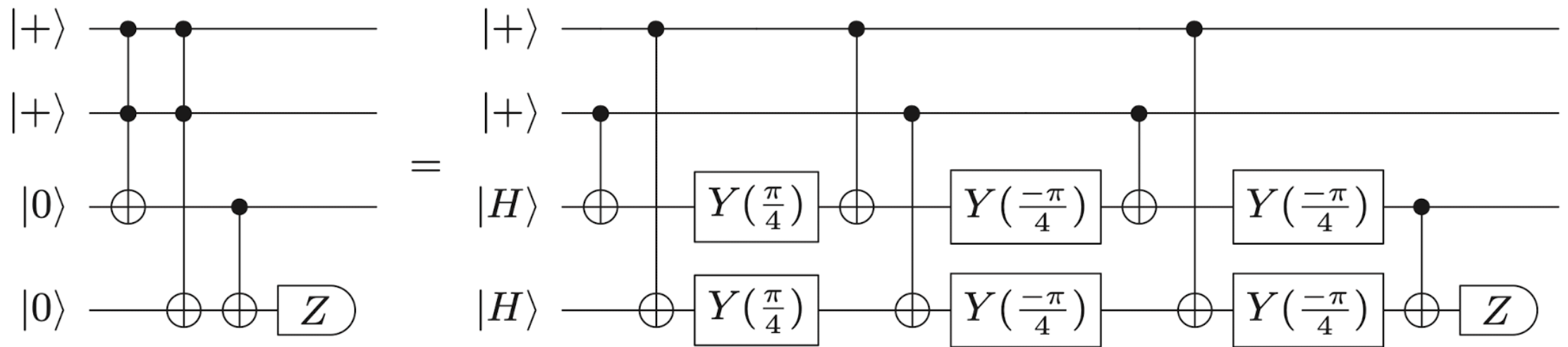


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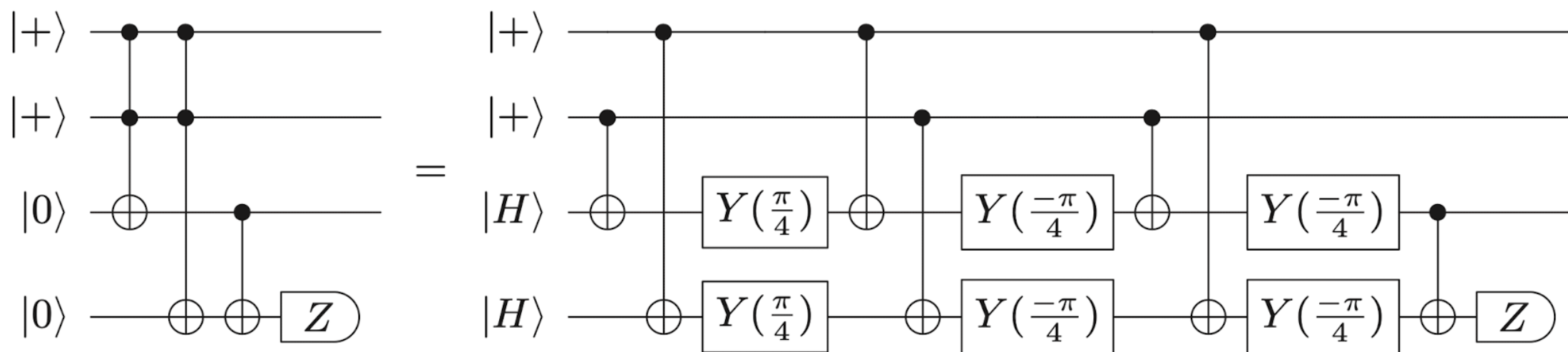


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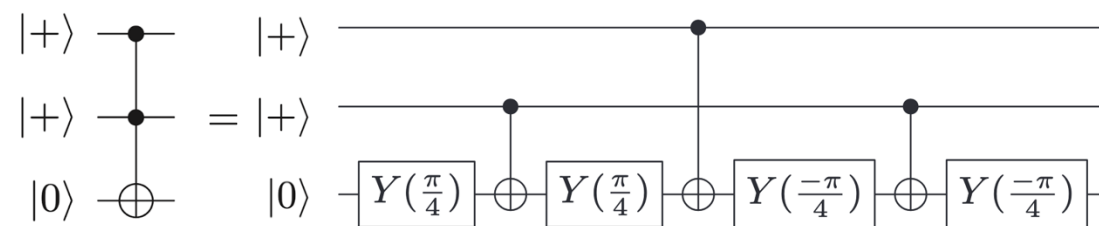


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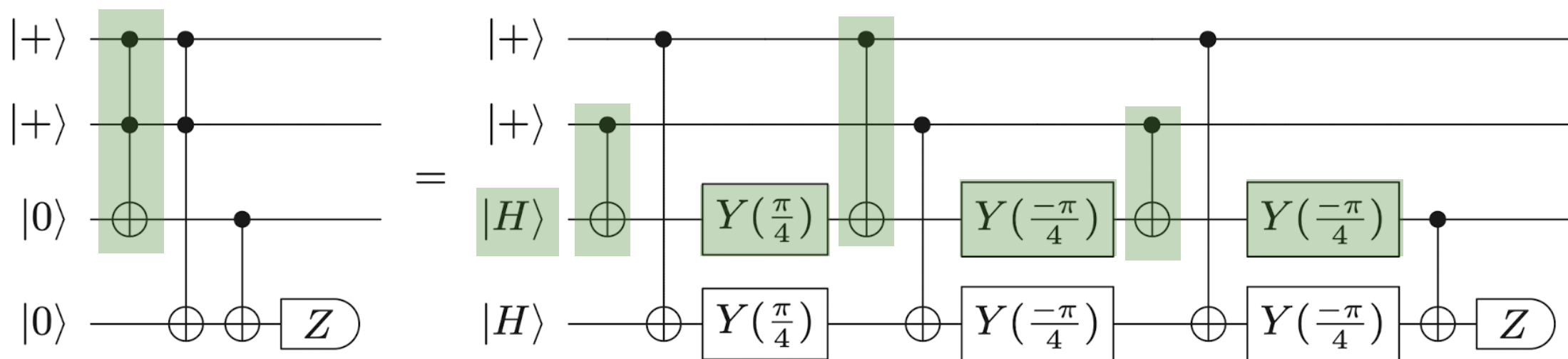


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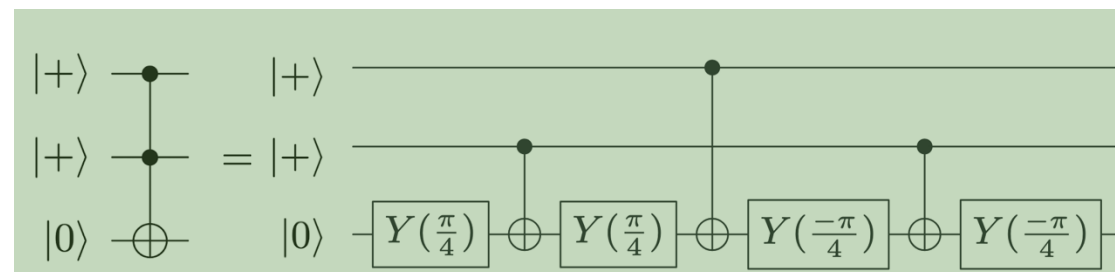


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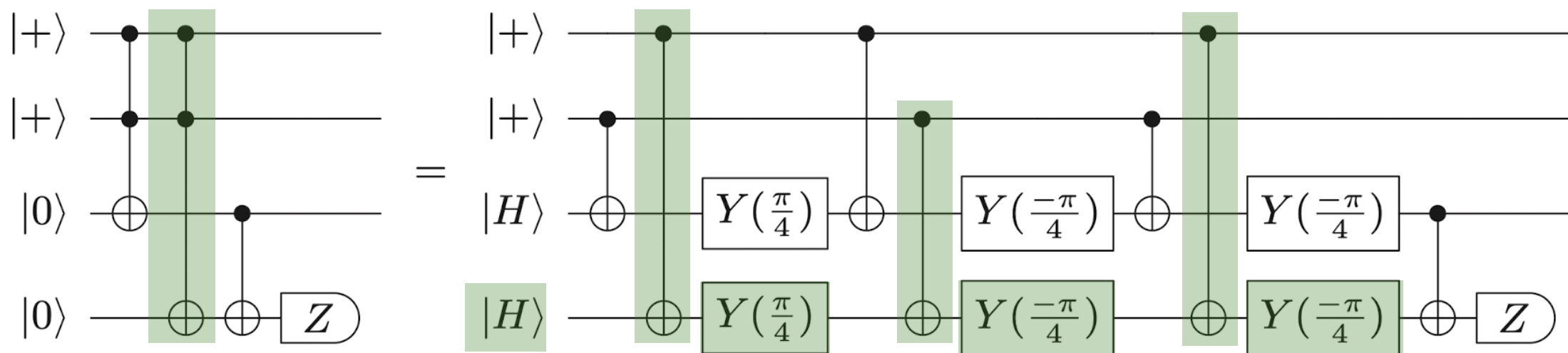


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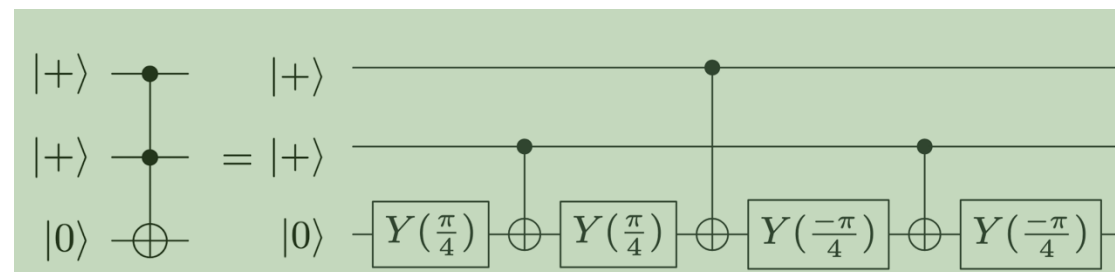


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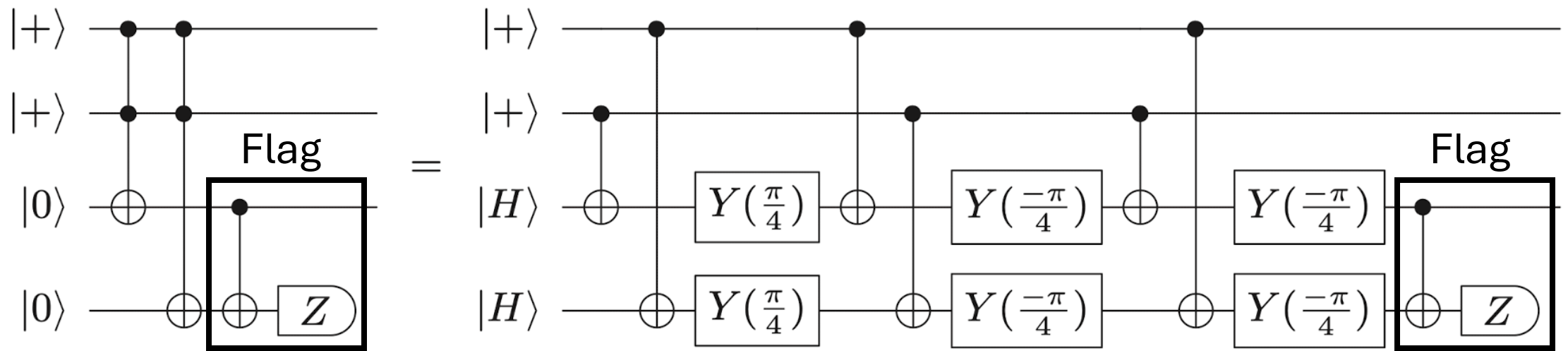


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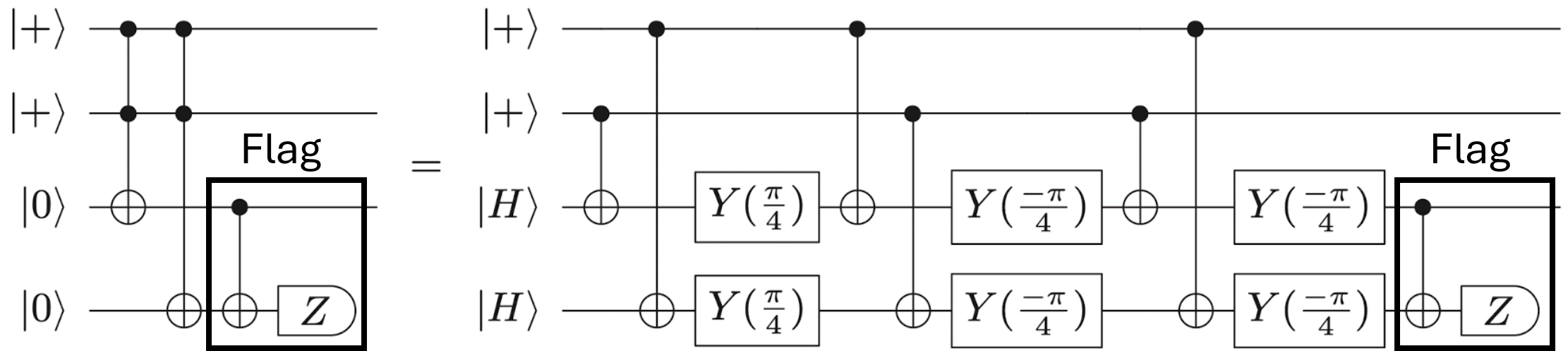


FIG. 1. H -to-Toffoli Distillation circuit. The output is discarded whenever a non-trivial outcome is obtained.

Output acceptance rate of the circuit is $a(p) = 1 - 8p + \mathcal{O}(p^2)$

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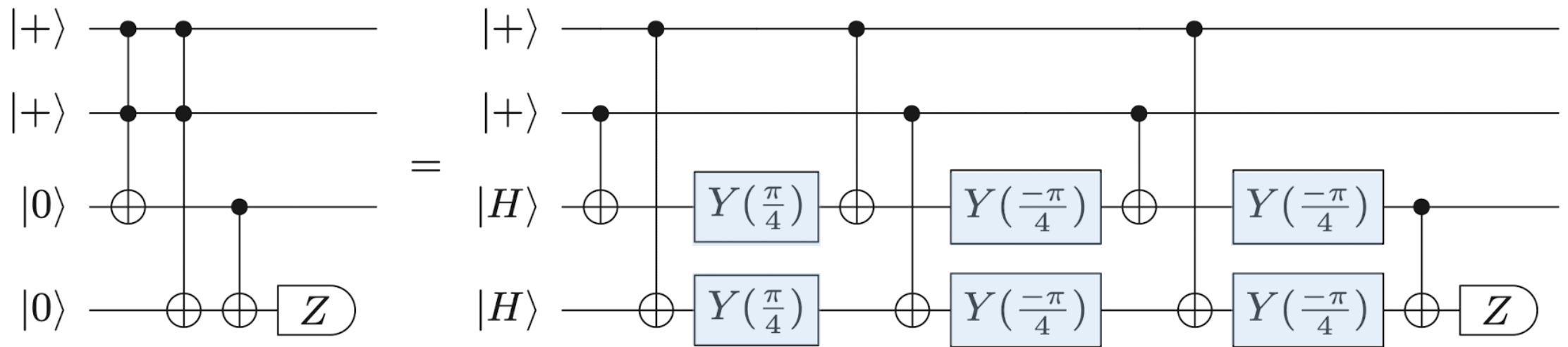


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But how do we realize
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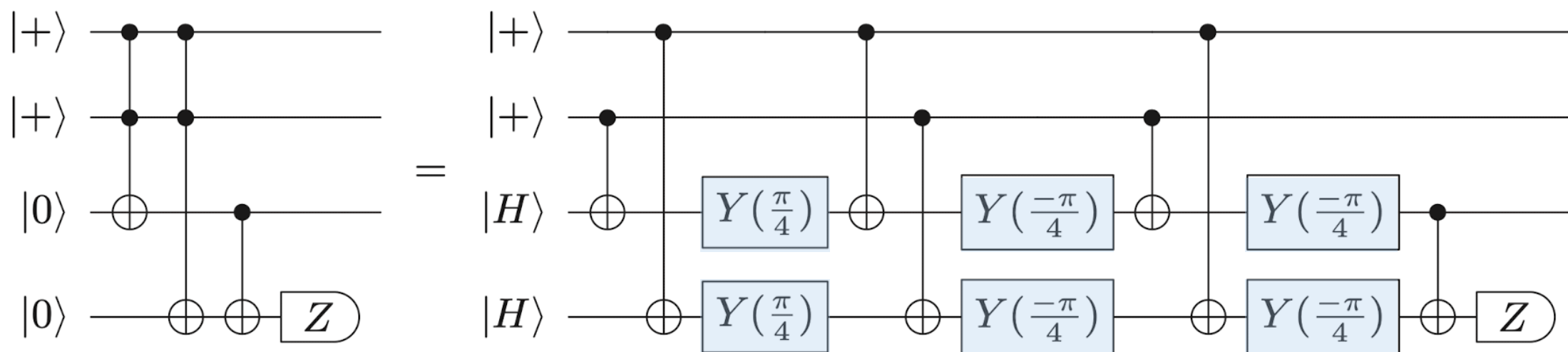


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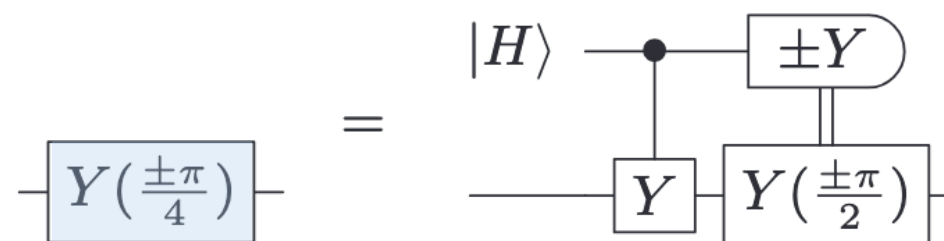


FIG. 4

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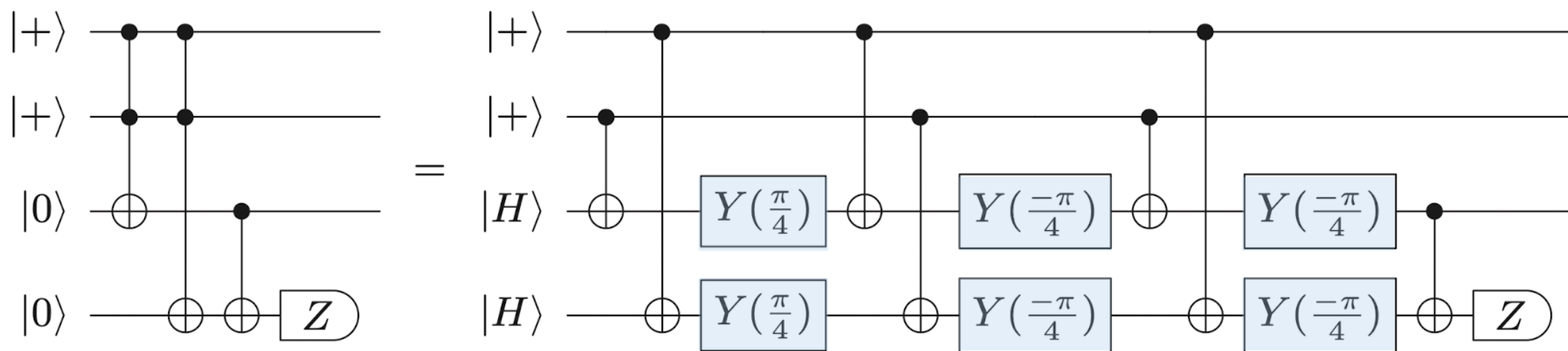


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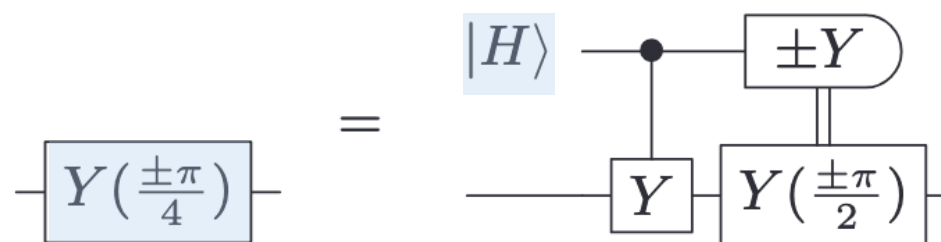


FIG. 4

Full Logical Distillation Circuit

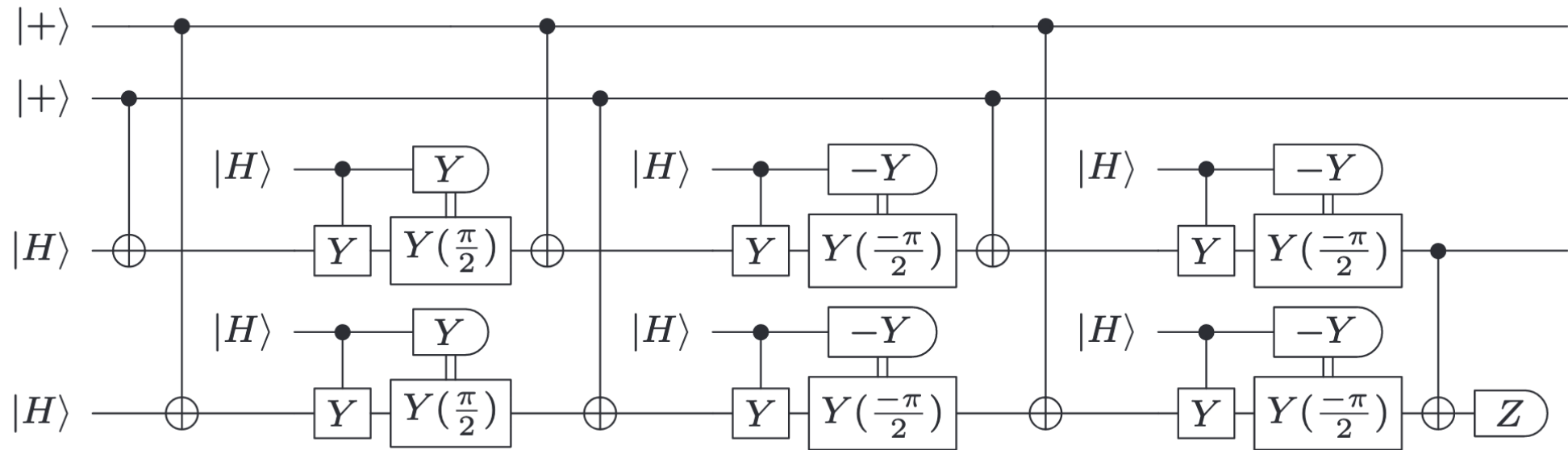


FIG. 5. Complete logical H -to-Toffoli distillation circuit.

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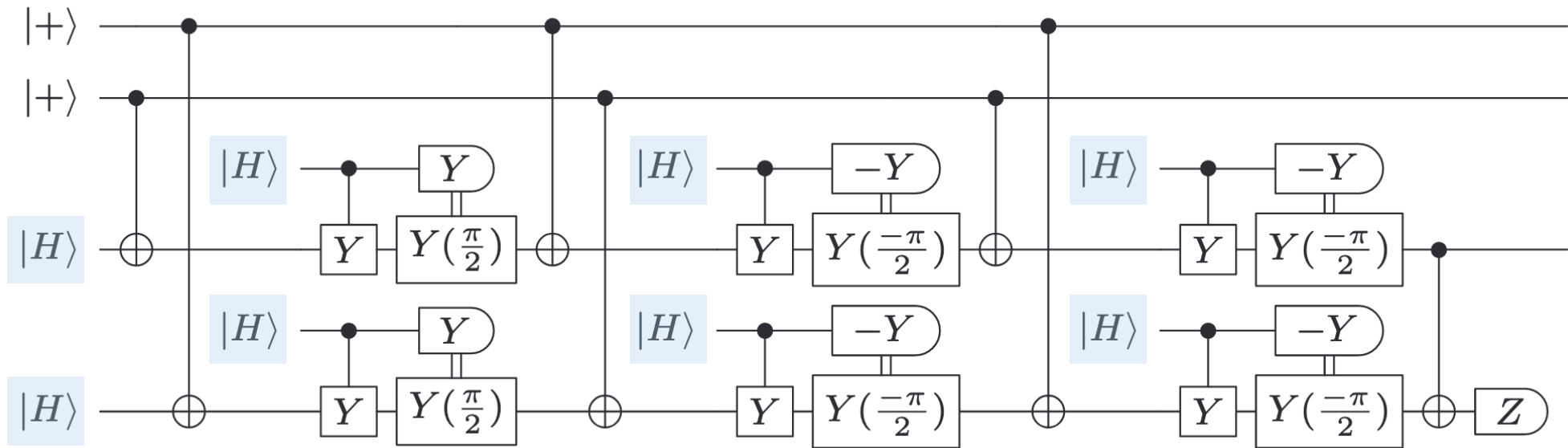


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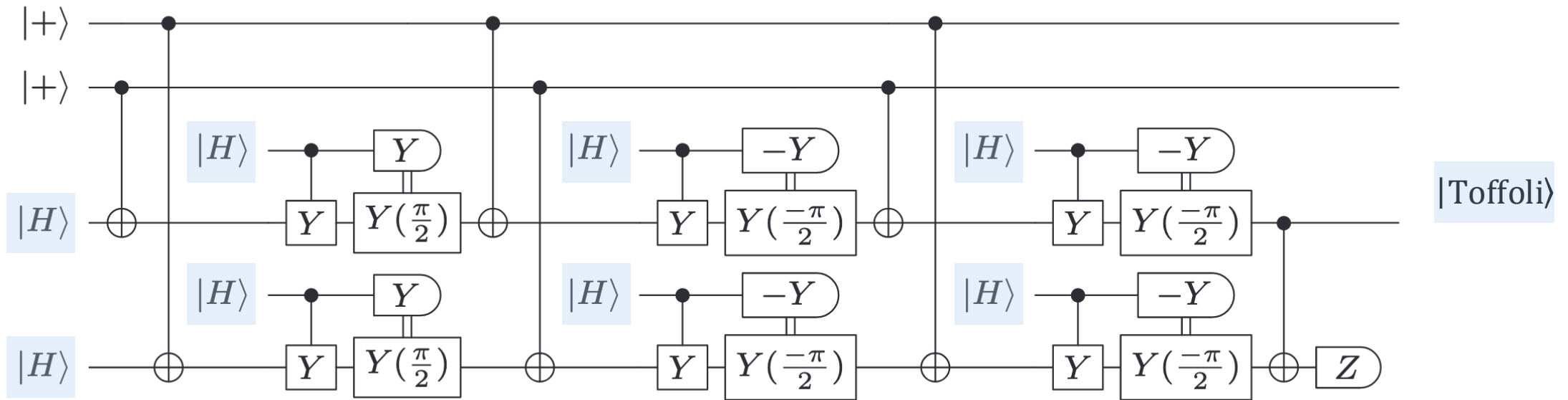


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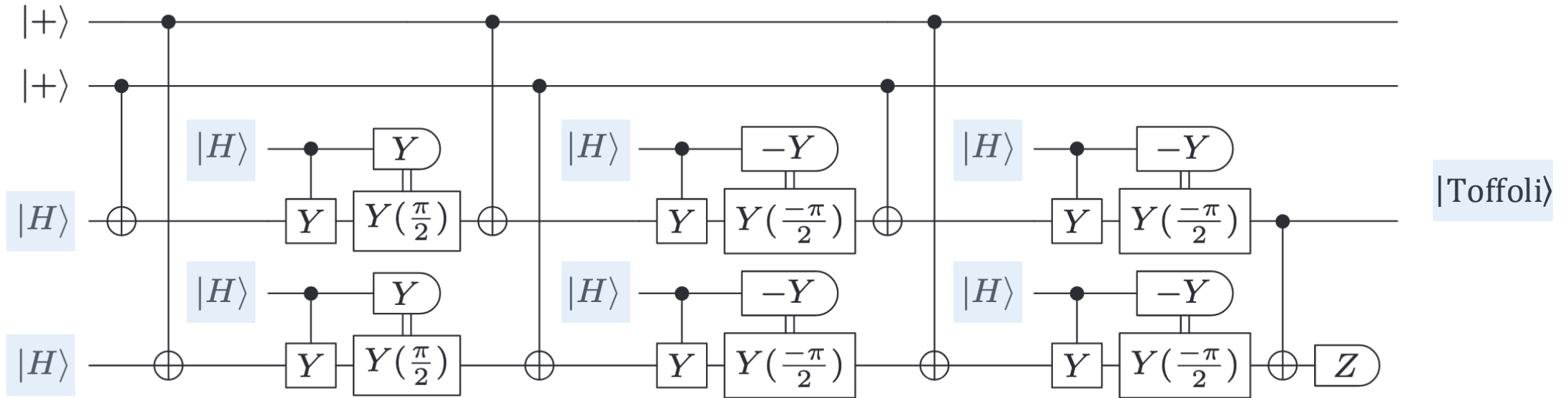


FIG. 5. Complete logical H -to-Toffoli distillation circuit.

Outputs $|Toffoli\rangle = (|000\rangle + |100\rangle + |010\rangle + |111\rangle)/2$
that can be injected to realize a logical Toffoli gate

Full Logical Distillation Circuit

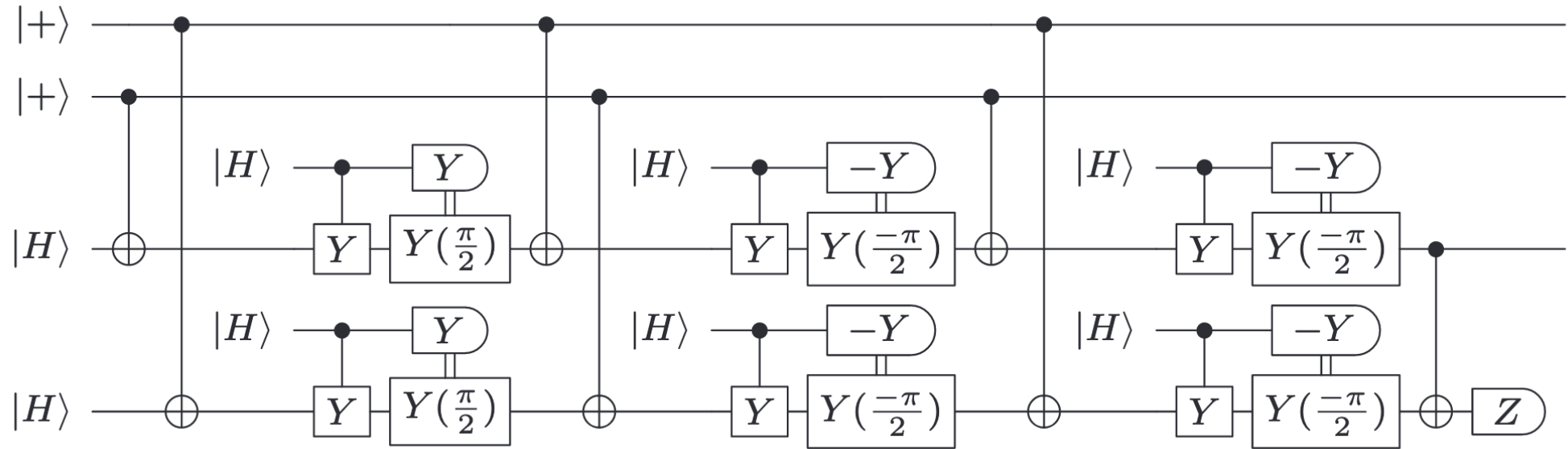
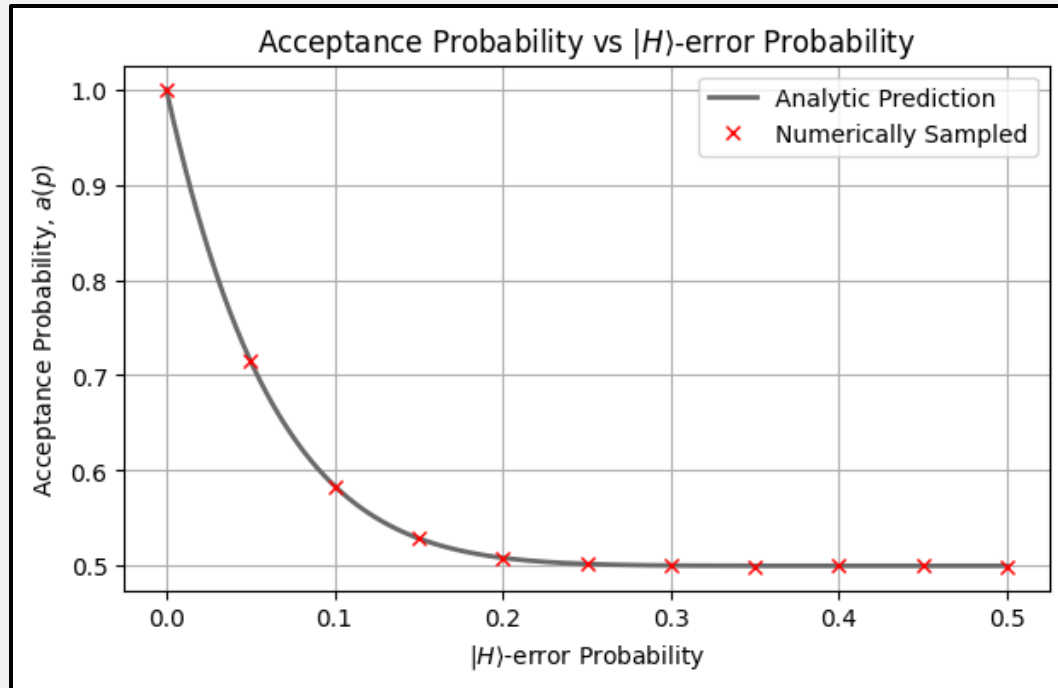


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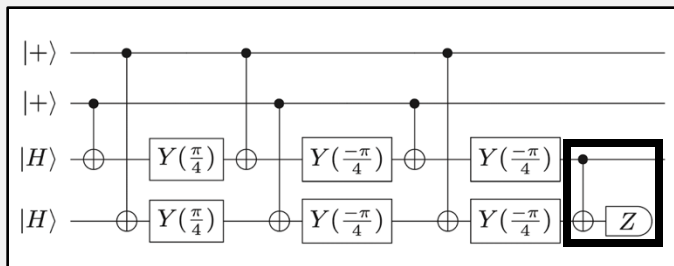
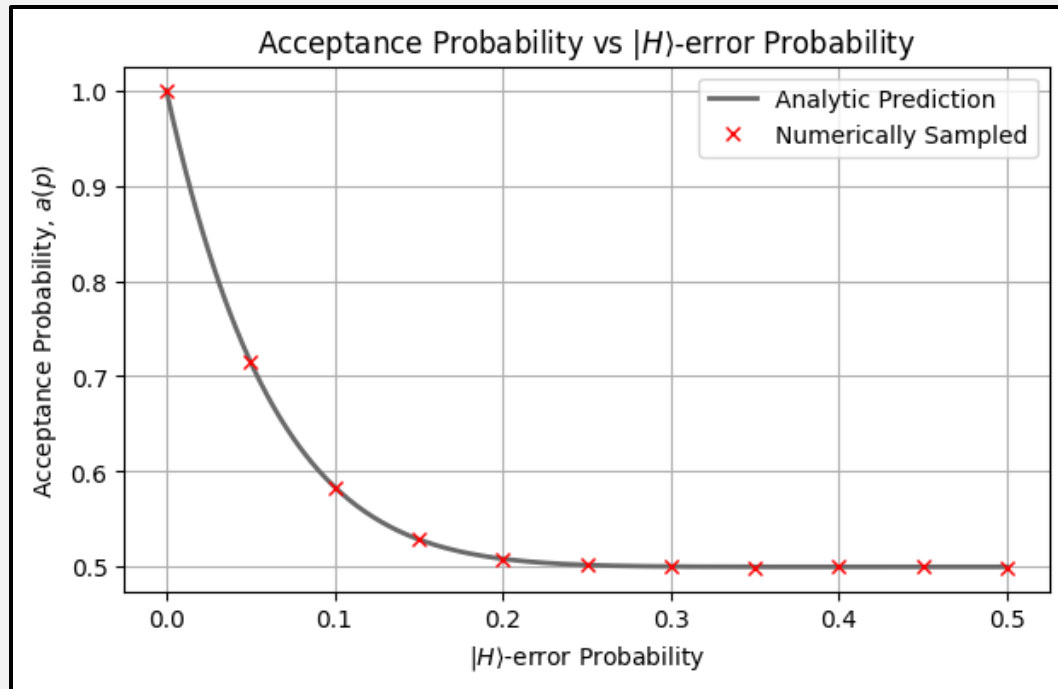
Reproducing $a(p)$, $e(p)a(p)$, for the Logical Circuit

Assuming perfect Cliffords!



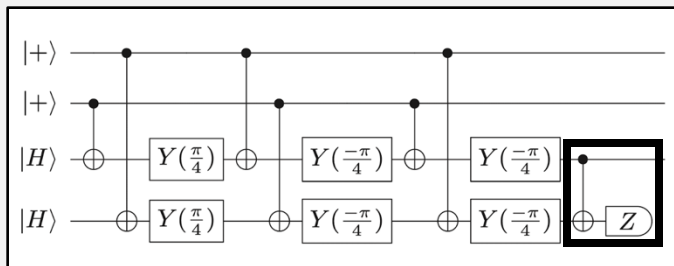
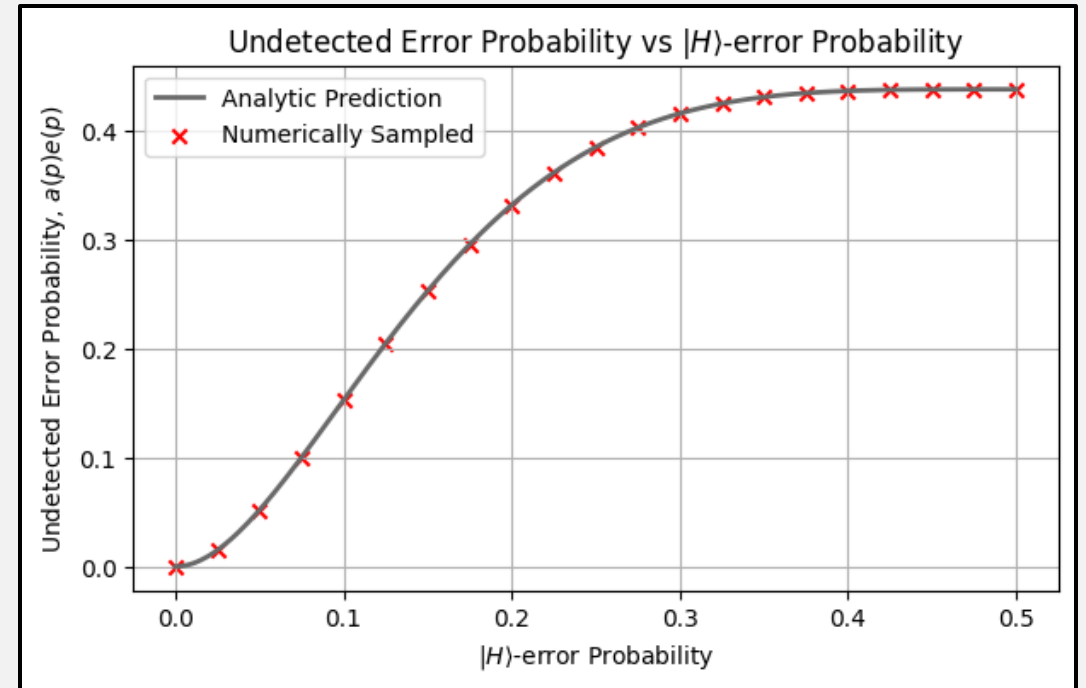
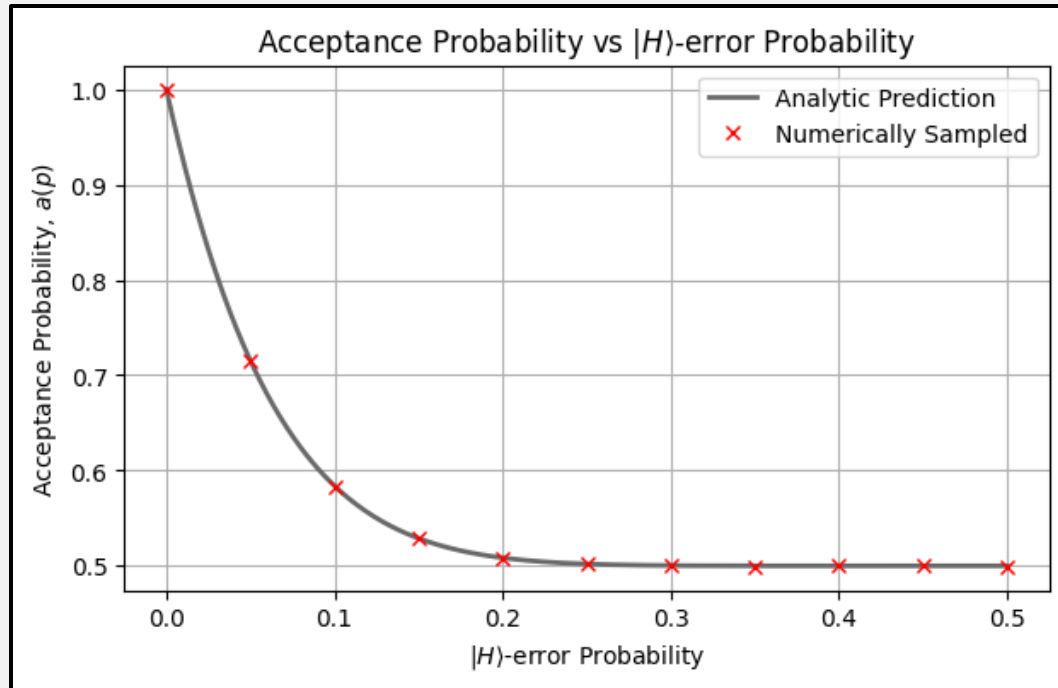
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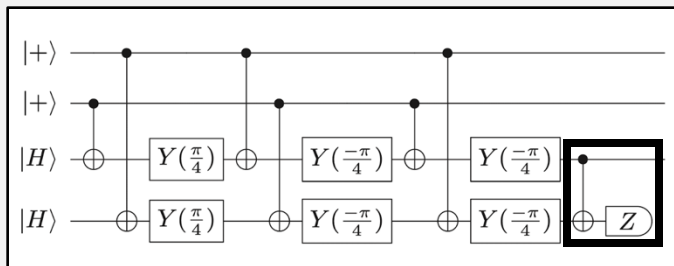
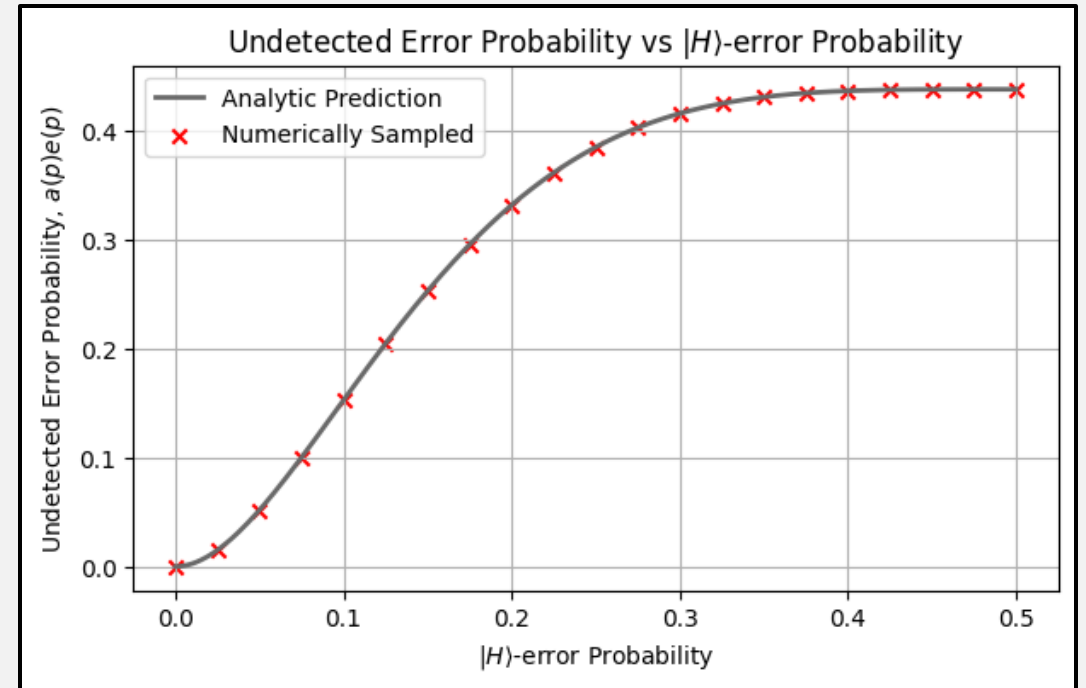
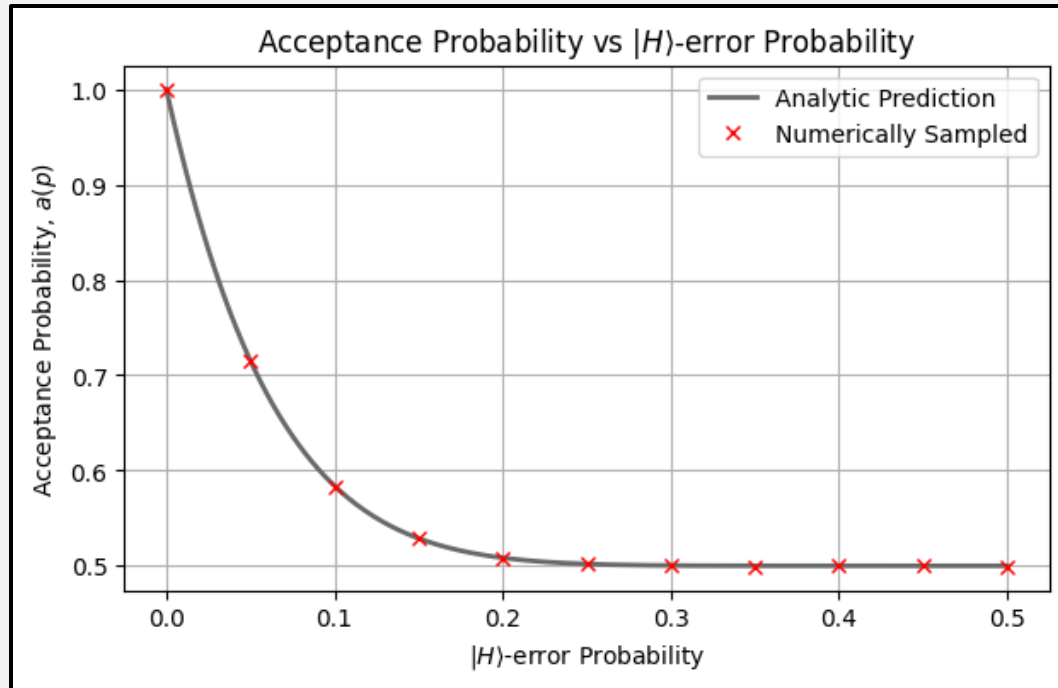
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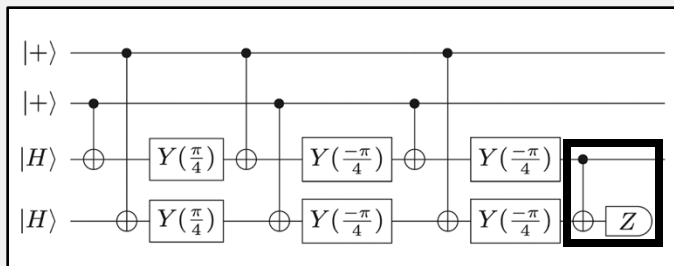
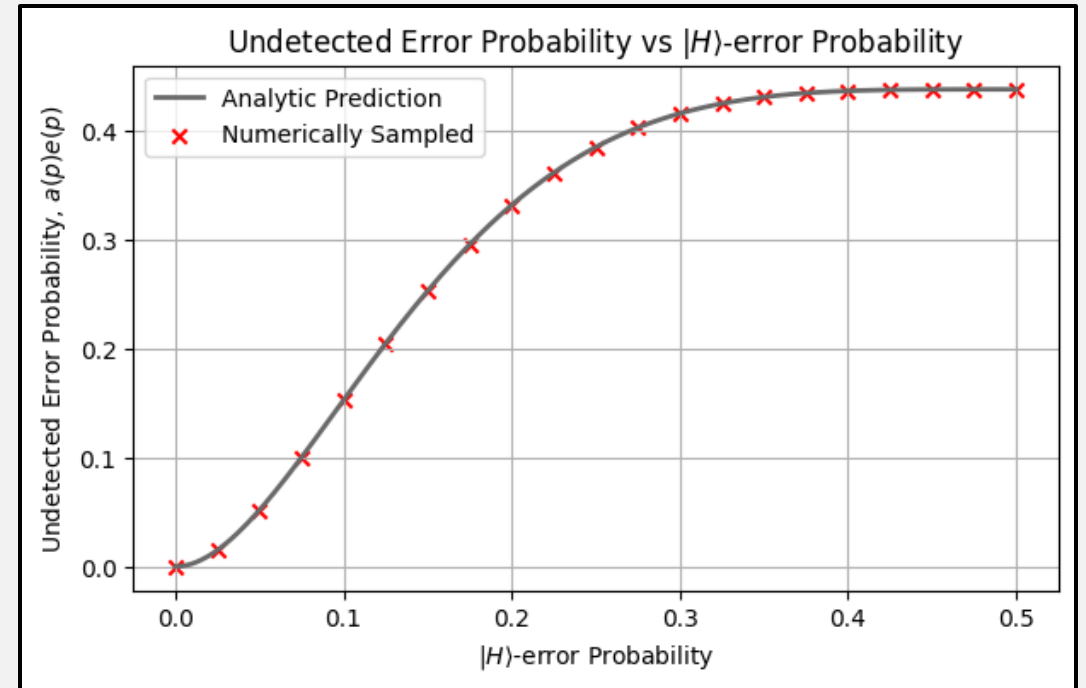
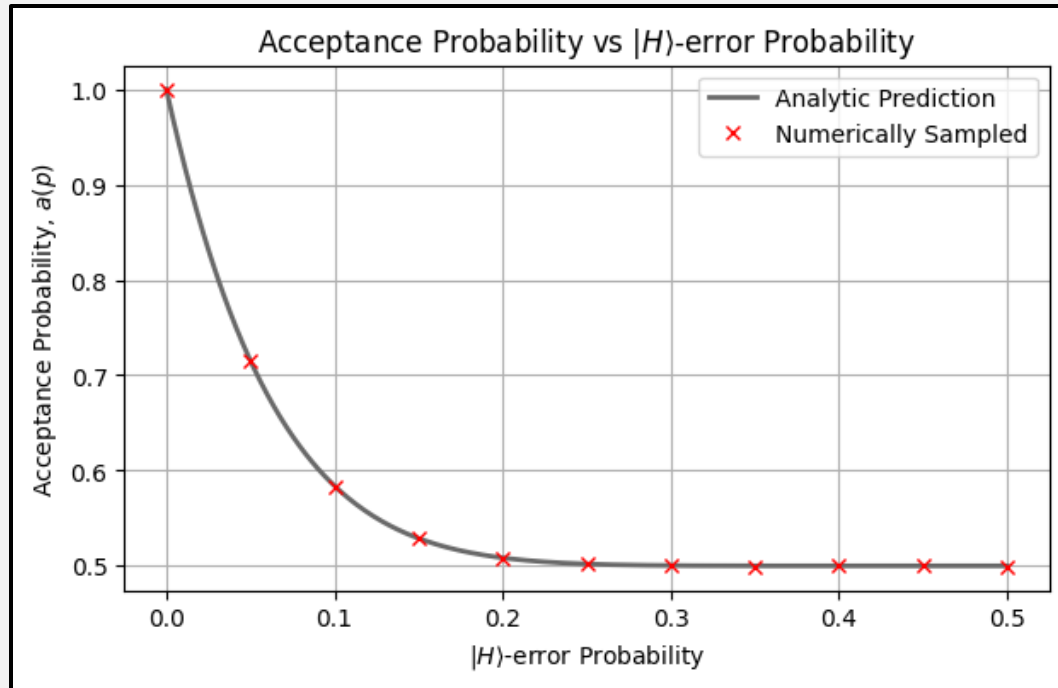
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Confirms our circuit is implemented correctly, but...

Reproducing $a(p)$, $e(p)a(p)$, for the Logical Circuit

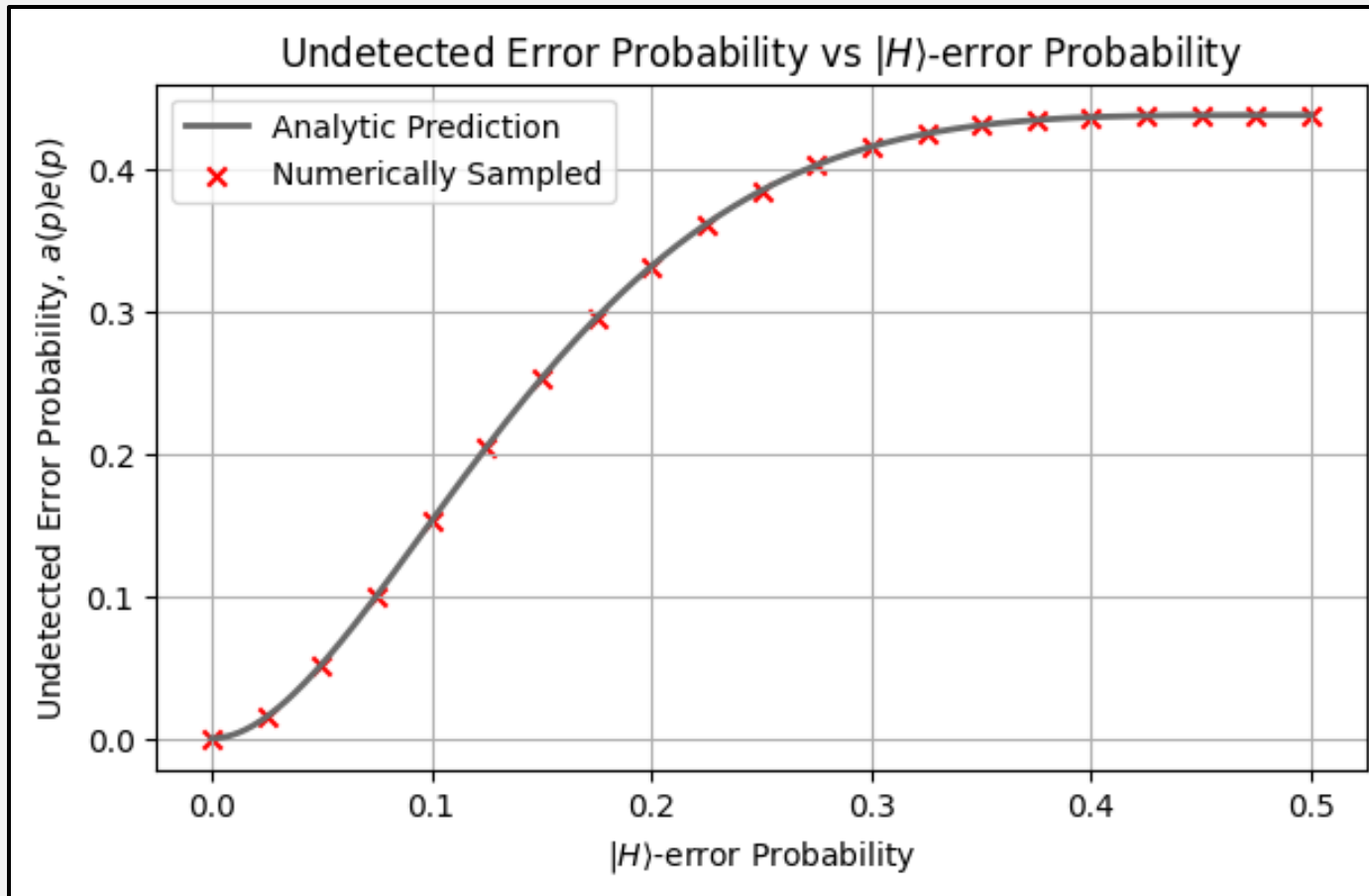
Assuming perfect Cliffords!



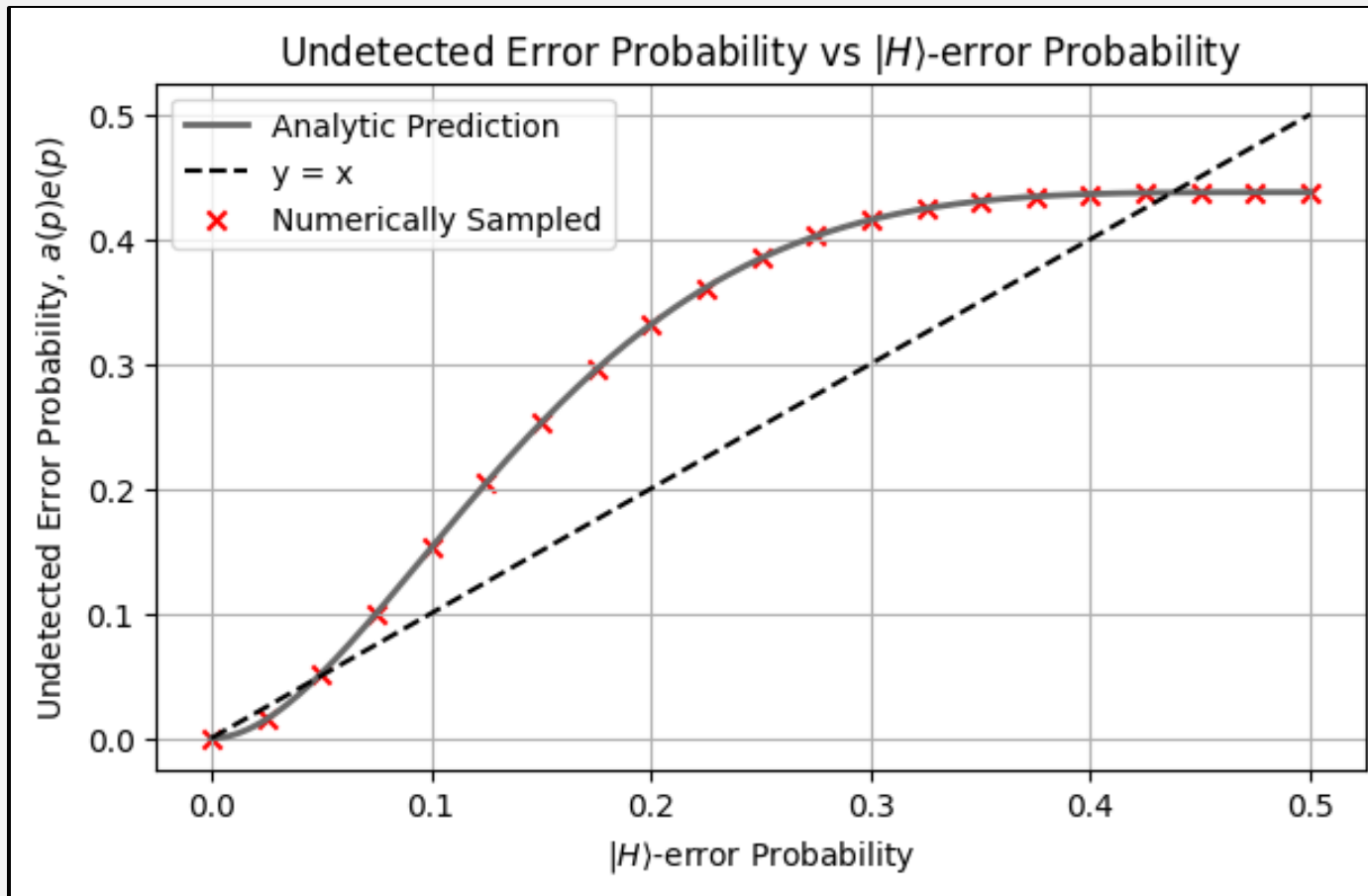
Confirms our circuit is implemented correctly, but...

when is it useful?

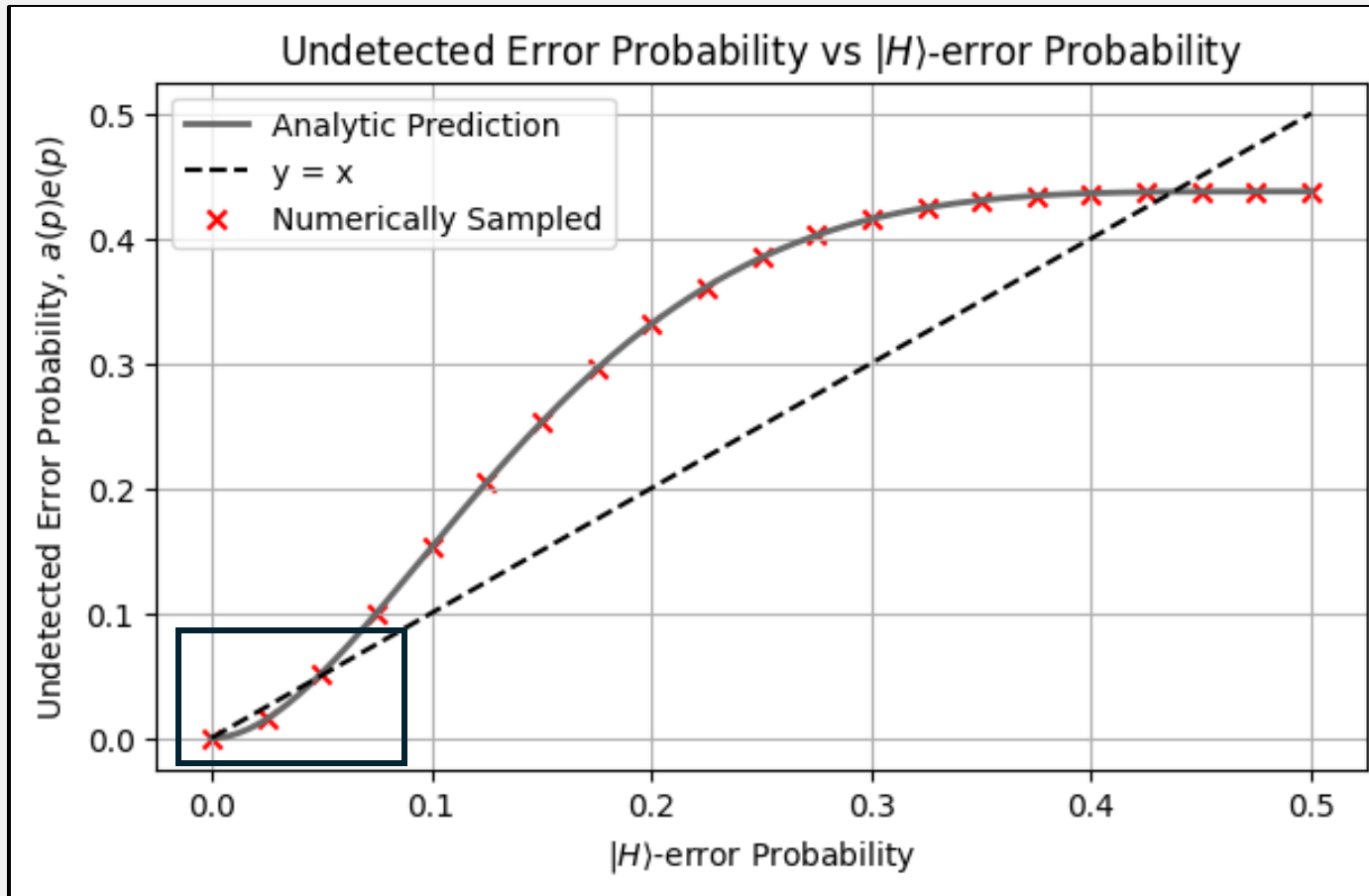
Undetected Error Probability, a Closer Examination



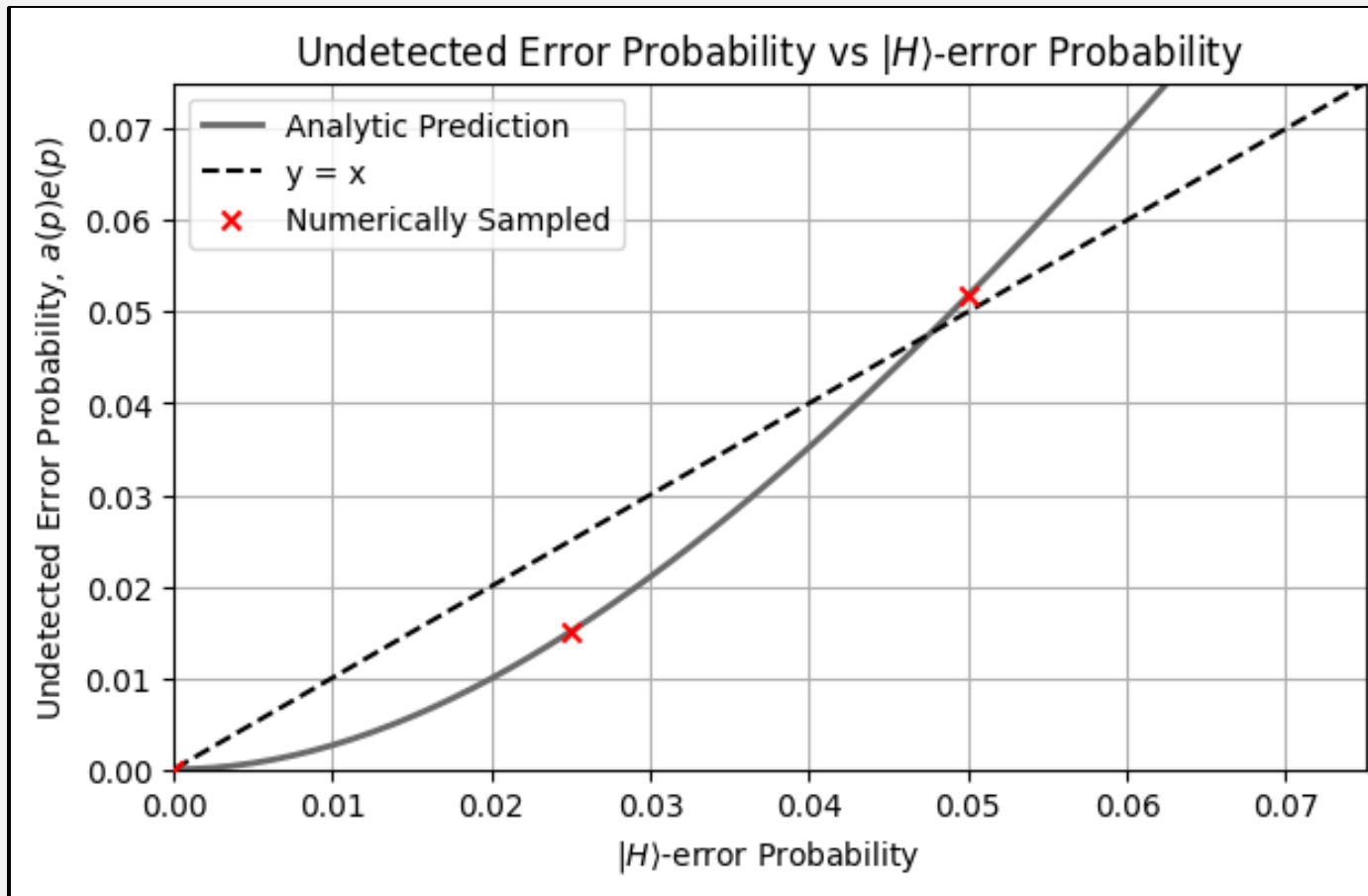
Undetected Error Probability, a Closer Examination



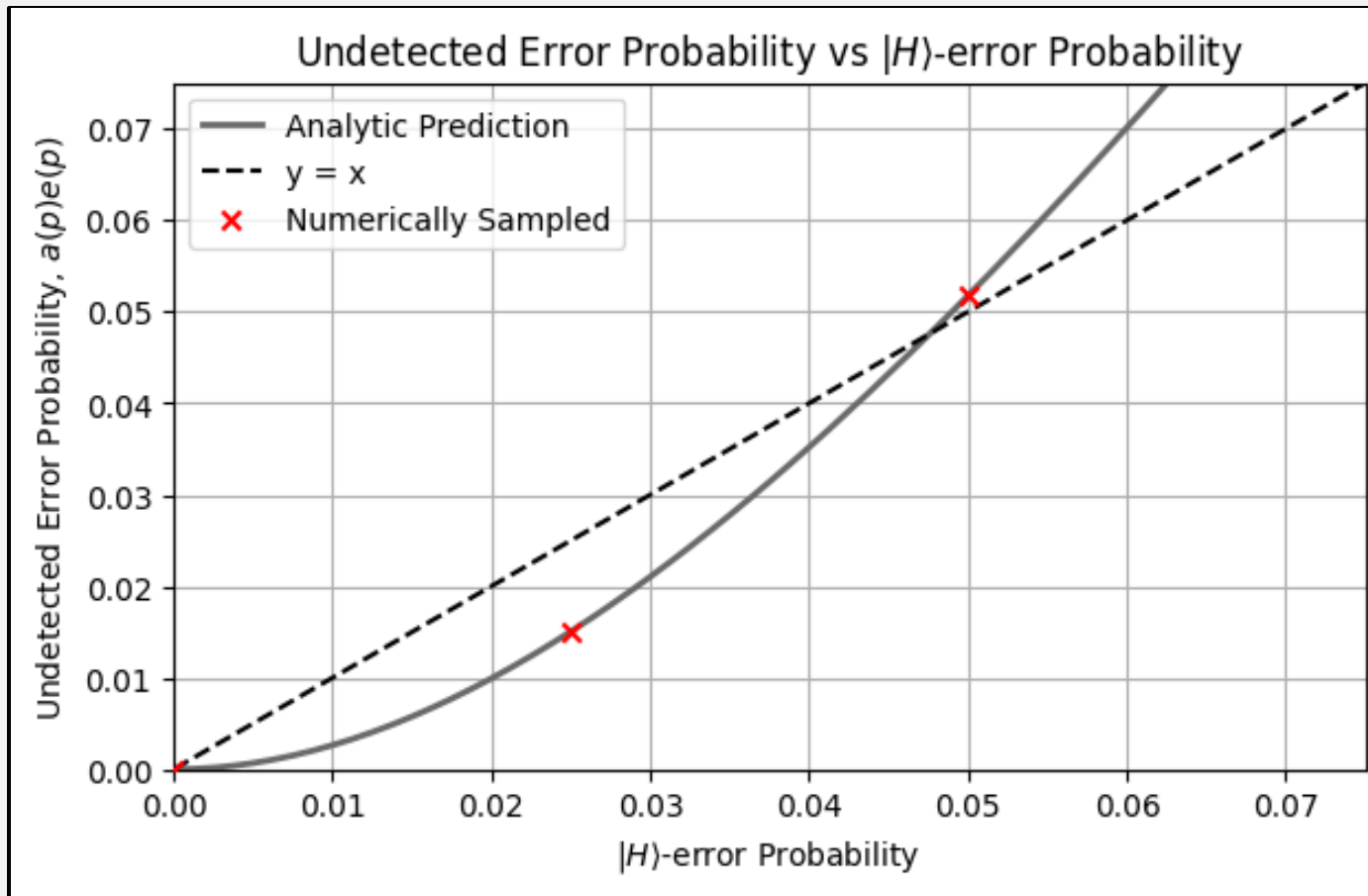
Undetected Error Probability, a Closer Examination



Undetected Error Probability, a Closer Examination

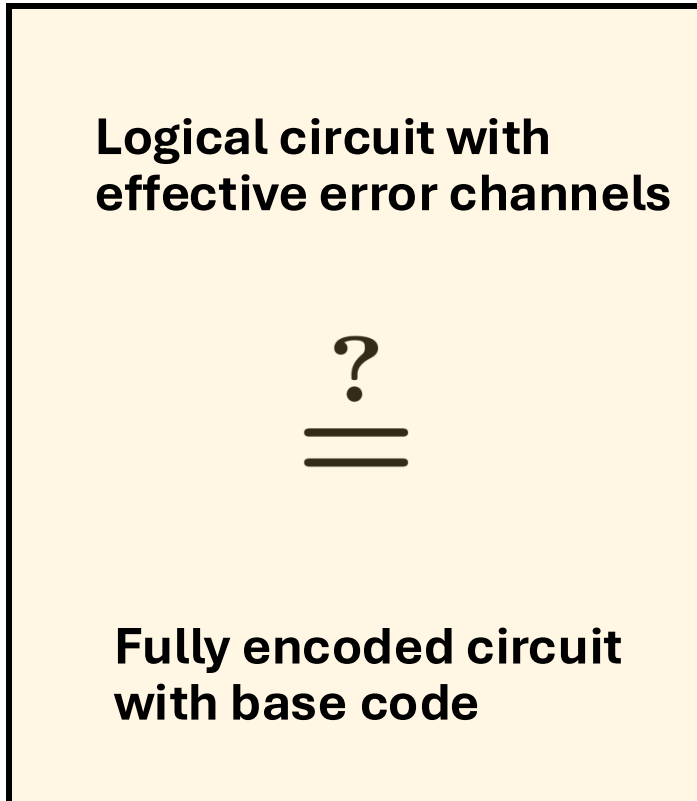


Undetected Error Probability, a Closer Examination

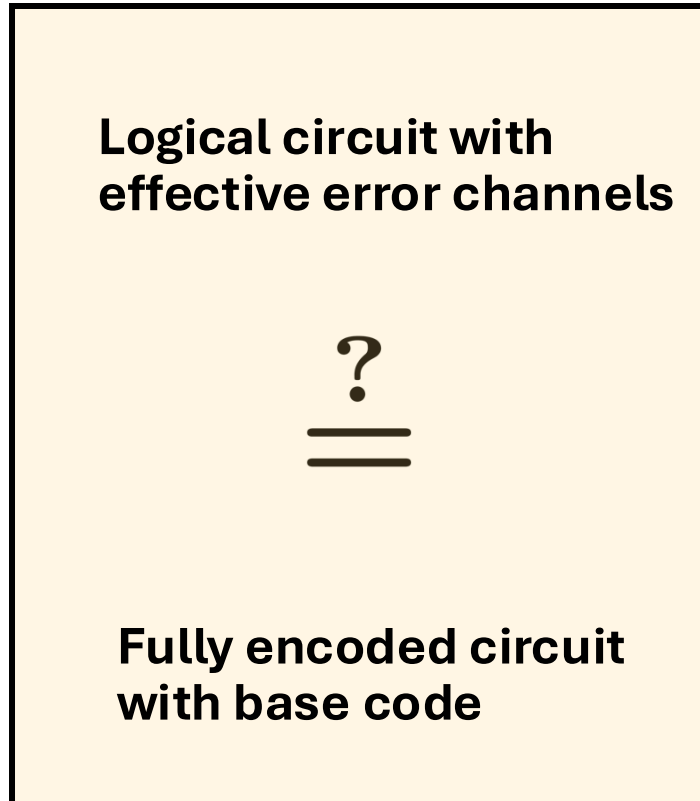


More on this later

Recall Research Question and Outline

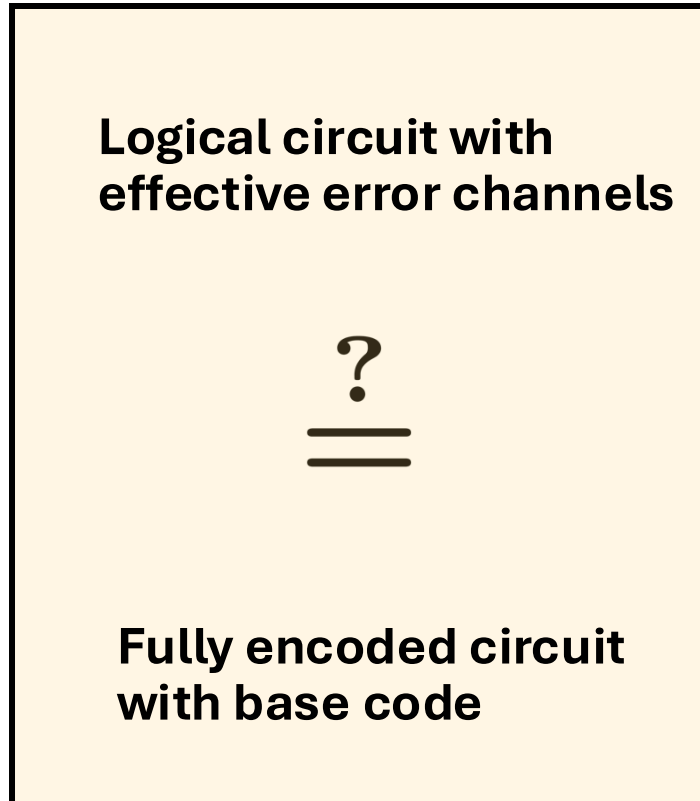


Recall Research Question and Outline



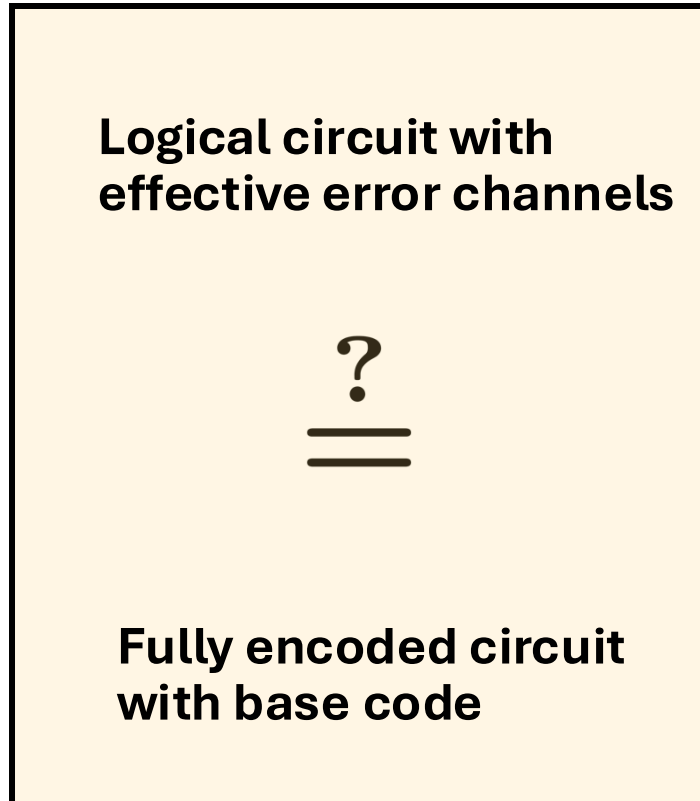
1. Logical circuit
2. Encoding our logical circuit
3. Constructing our effective error channels

Recall Research Question and Outline



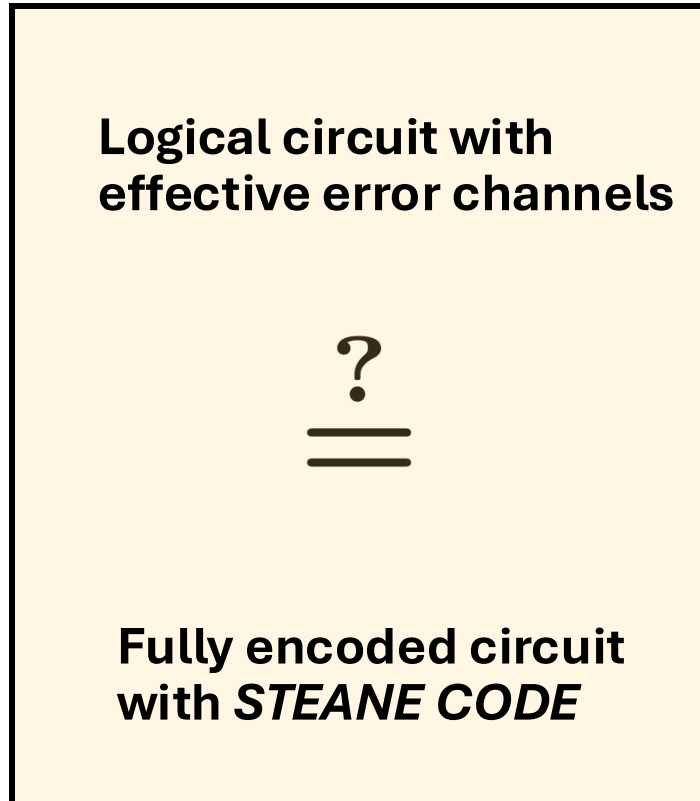
- ~~1. Logical circuit~~
2. Encoding our logical circuit
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Recall Research Question and Outline



- ~~1. Logical circuit~~
- 2. Encoding our logical circuit**
3. Constructing our effective error channels

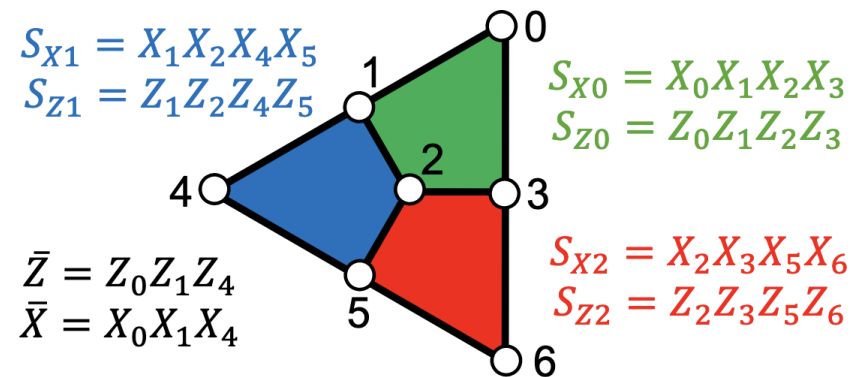
Recall Research Question and Outline



- ~~1. Logical circuit~~
- 2. Encoding our logical circuit**
3. Constructing our effective error channels

Steane-encoding

- The Steane Code
 - Detects and corrects all weight-1 errors
 - All Clifford gates can be implemented transversally



Full Steane-encoded Circuit

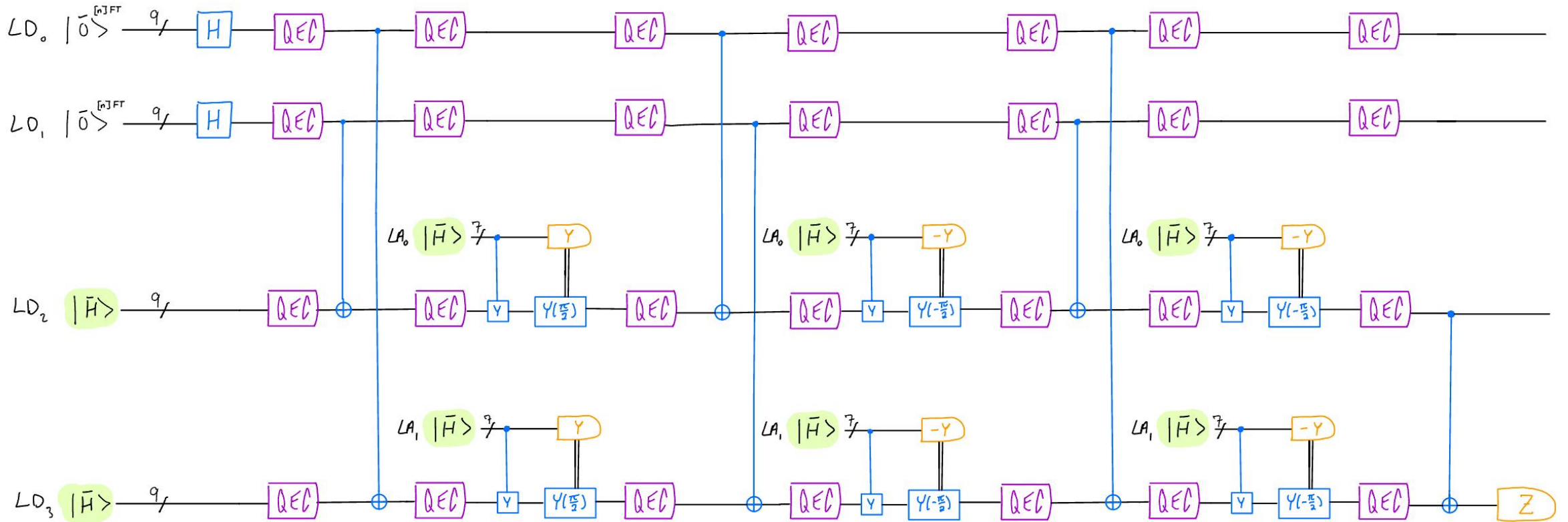
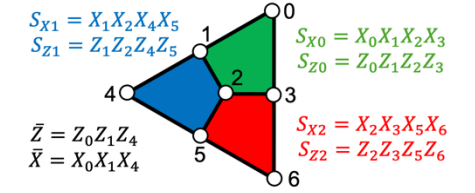


FIG. 6. Complete Steane-encoded $|H\rangle$ - $|Toffoli\rangle$ distillation circuit we want to simulate.

II. Effective Error Channels

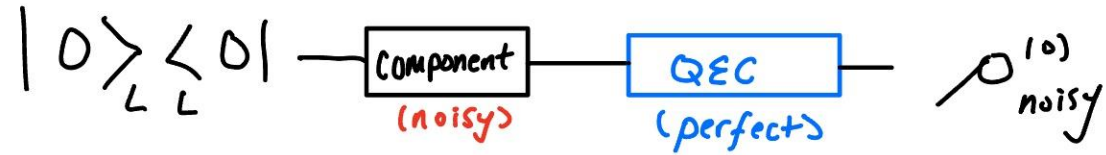
Process Tomography:

A method to reconstruct a full error channel describing the noise of a component under an arbitrary input state

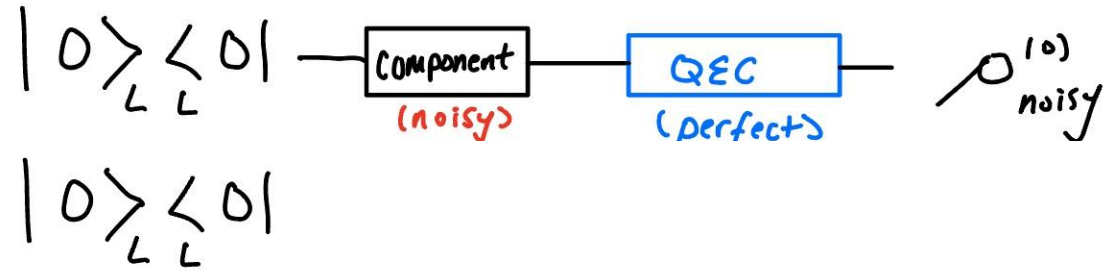
One-Qubit Process Tomography

$$|0\rangle_L \langle 0|$$

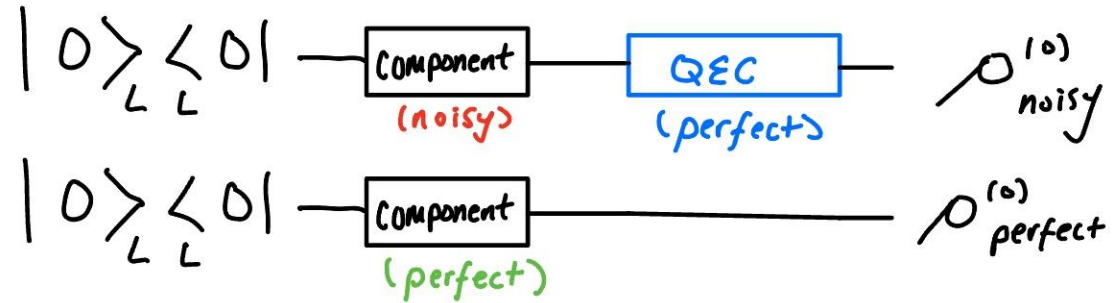
One-Qubit Process Tomography



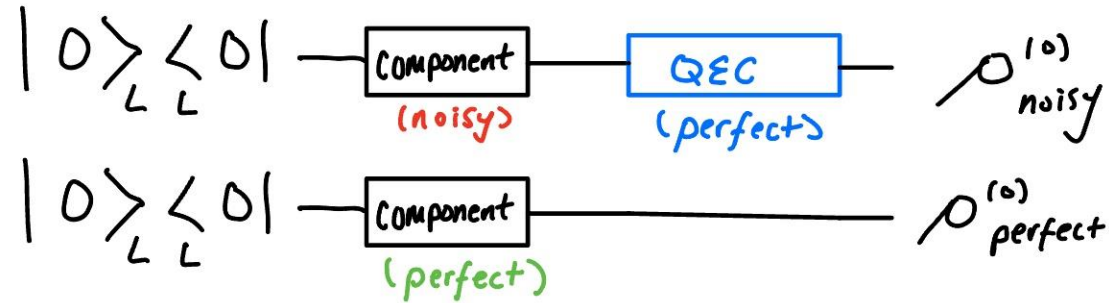
One-Qubit Process Tomography



One-Qubit Process Tomography

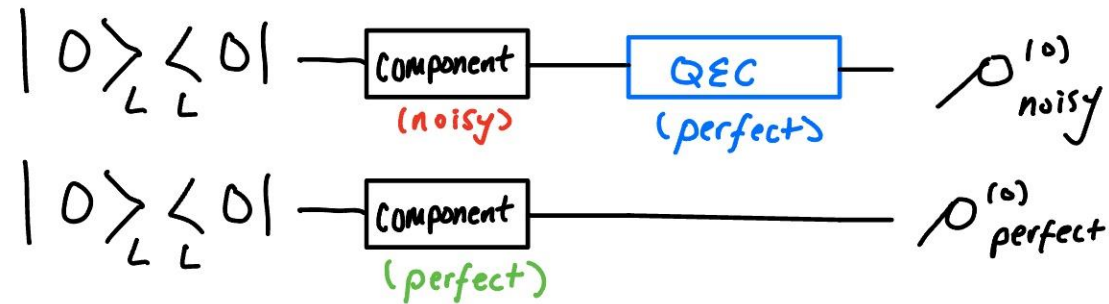


One-Qubit Process Tomography



$$1 - F[\rho^{(0)}_{\text{noisy}}, \rho^{(0)}_{\text{perfect}}] = p_x + p_y$$

One-Qubit Process Tomography



$$1 - F[\rho^{(0)}_{\text{noisy}}, \rho^{(0)}_{\text{perfect}}] = p_x + p_y$$

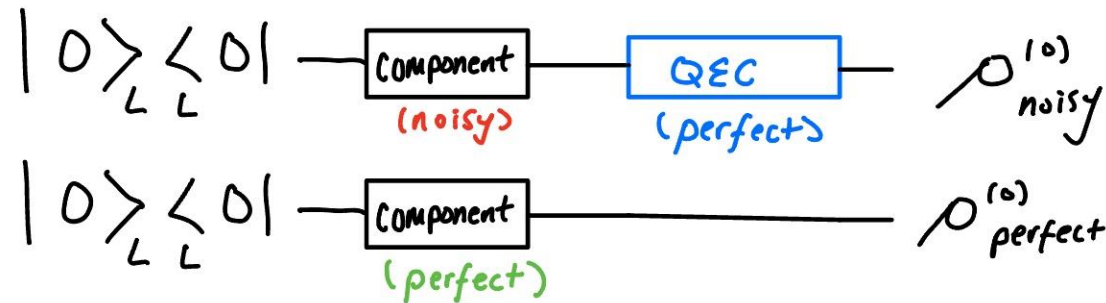
$$|+\rangle_L \langle_L +|$$

$$|+\rangle_L \langle_L +|$$

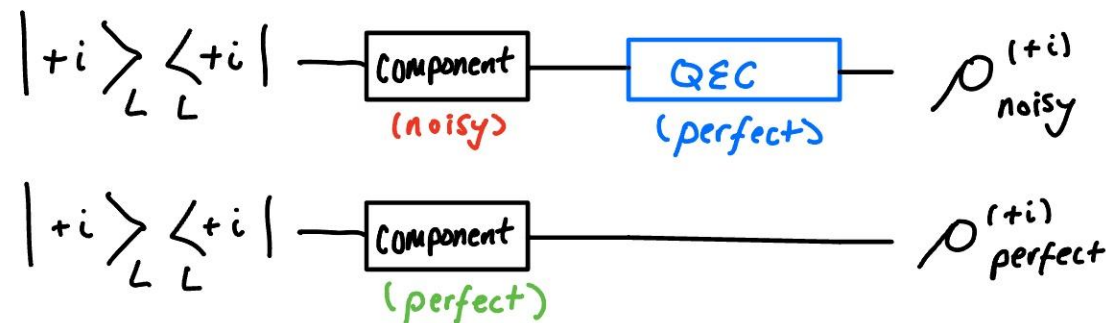
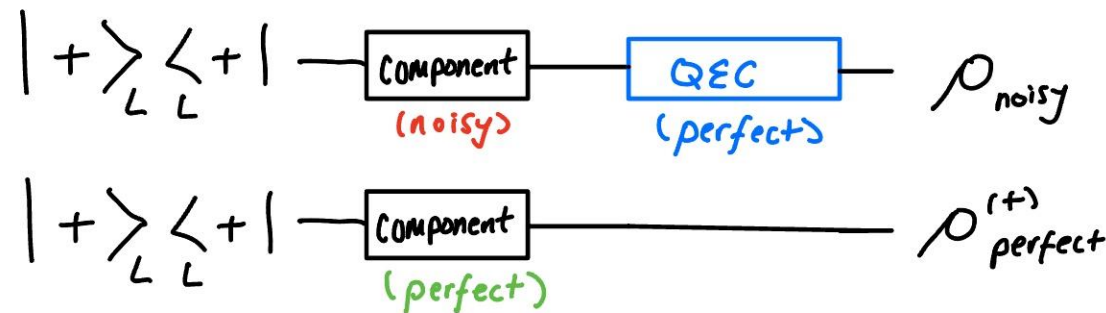
$$|+i\rangle_L \langle_L +i|$$

$$|+i\rangle_L \langle_L +i|$$

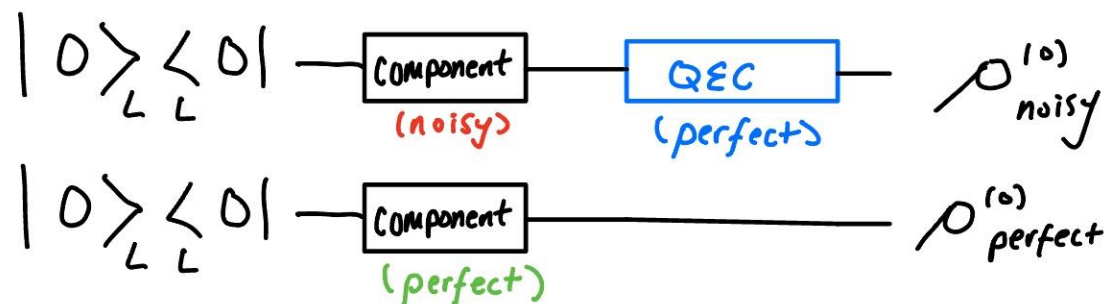
One-Qubit Process Tomography



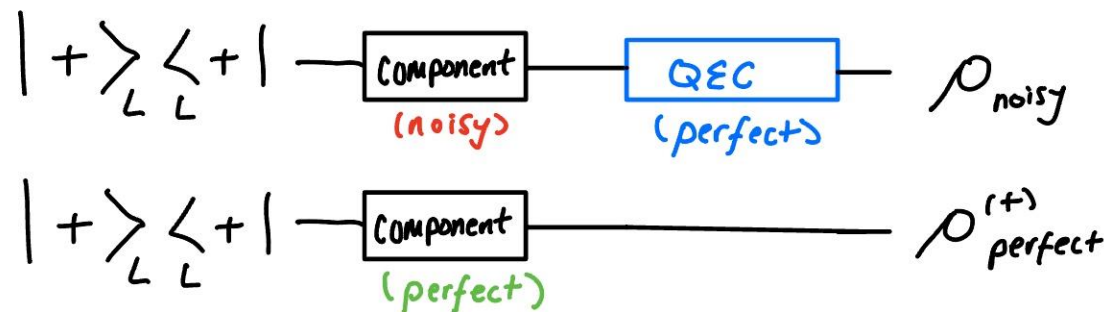
$$1 - F[\rho_{\text{noisy}}^{(0)}, \rho_{\text{perfect}}^{(0)}] = p_x + p_y$$



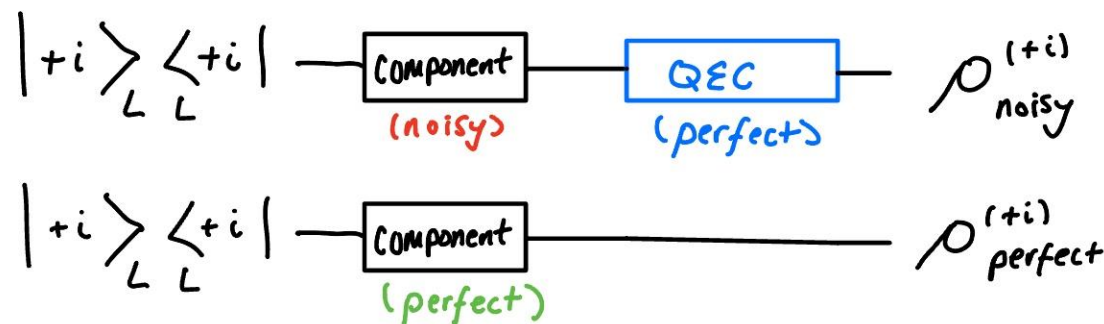
One-Qubit Process Tomography



$$1 - F[\rho_{\text{noisy}}^{(0)}, \rho_{\text{perfect}}^{(0)}] = p_x + p_y$$

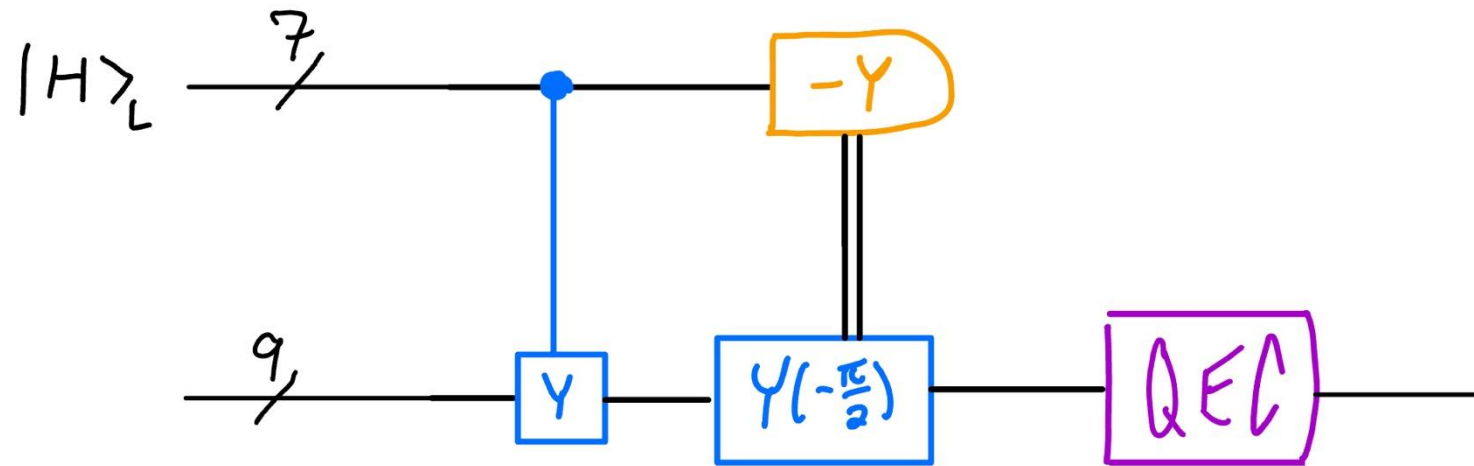


$$1 - F[\rho_{\text{noisy}}^{(+)}, \rho_{\text{perfect}}^{(+)}] = p_y + p_z$$

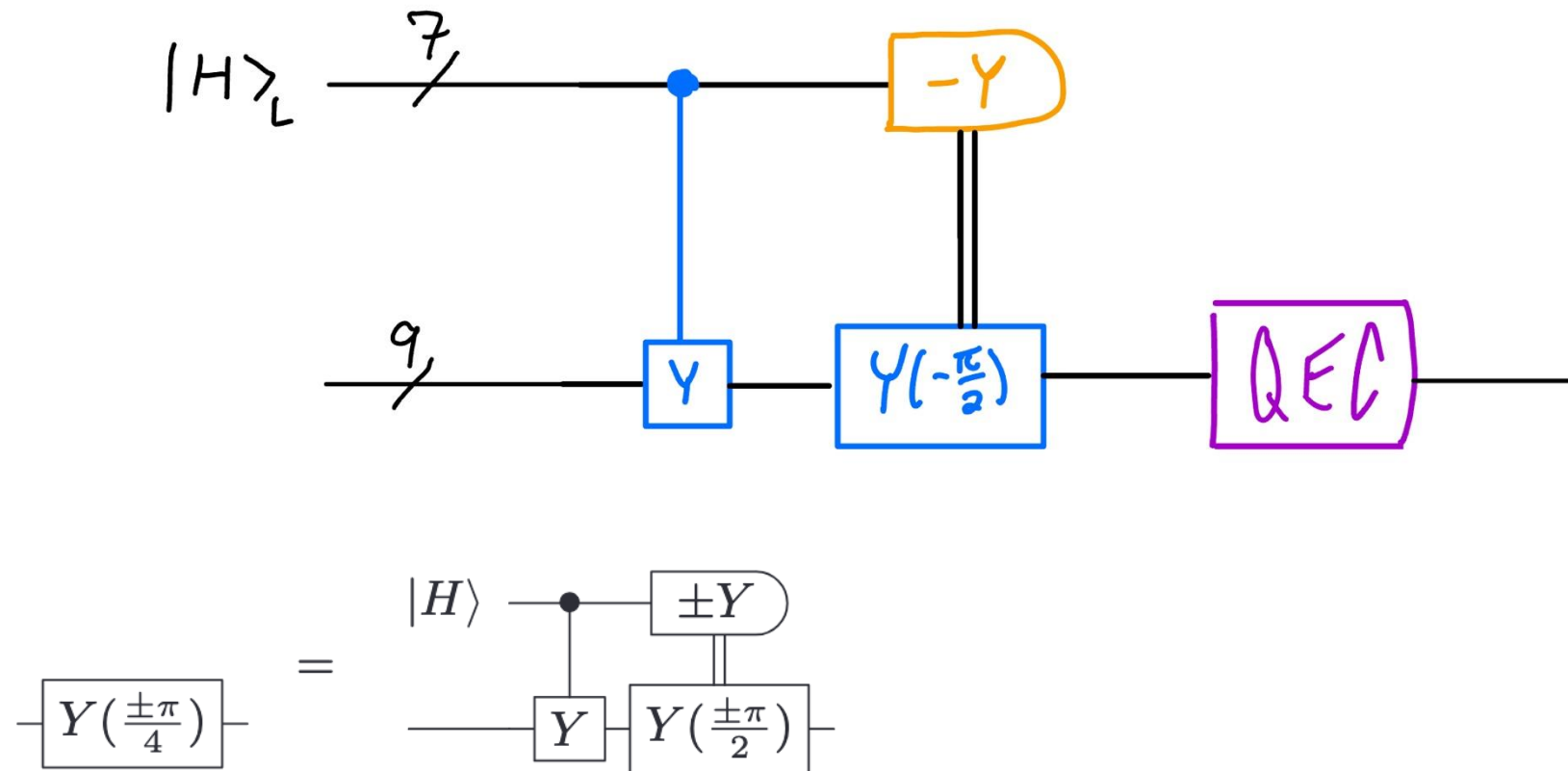


$$1 - F[\rho_{\text{noisy}}^{(+i)}, \rho_{\text{perfect}}^{(+i)}] = p_x + p_z$$

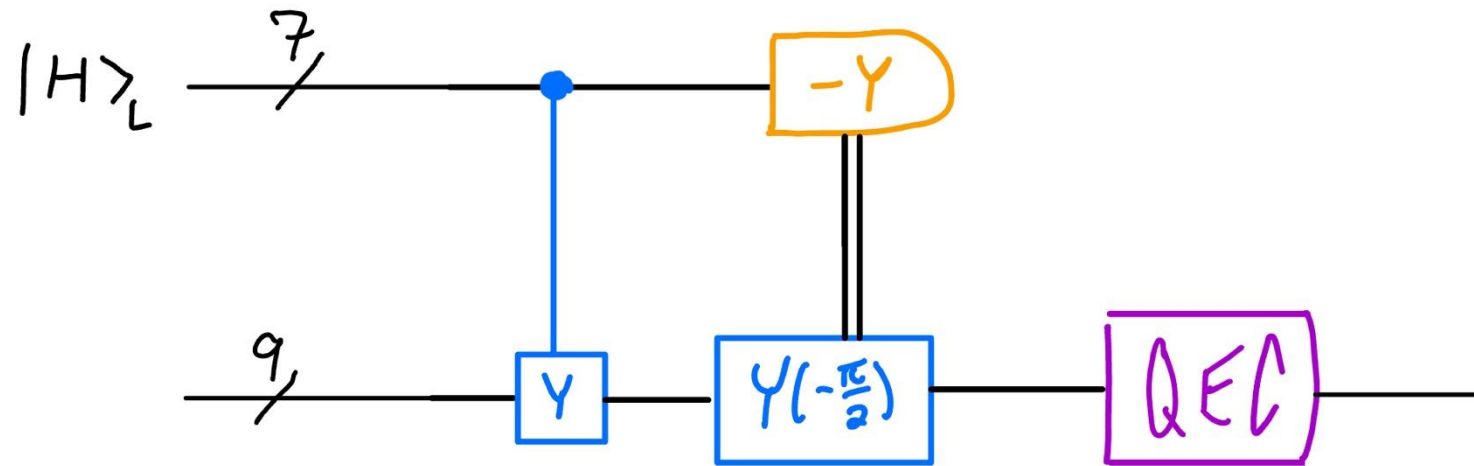
Logical Injection Circuit Tomography



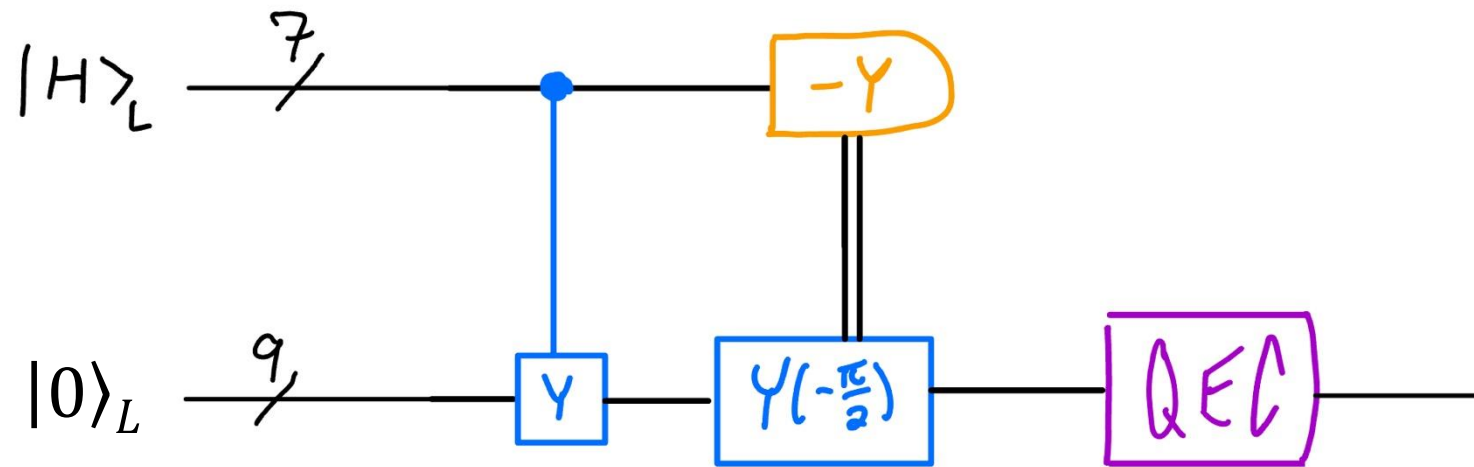
Logical Injection Circuit Tomography



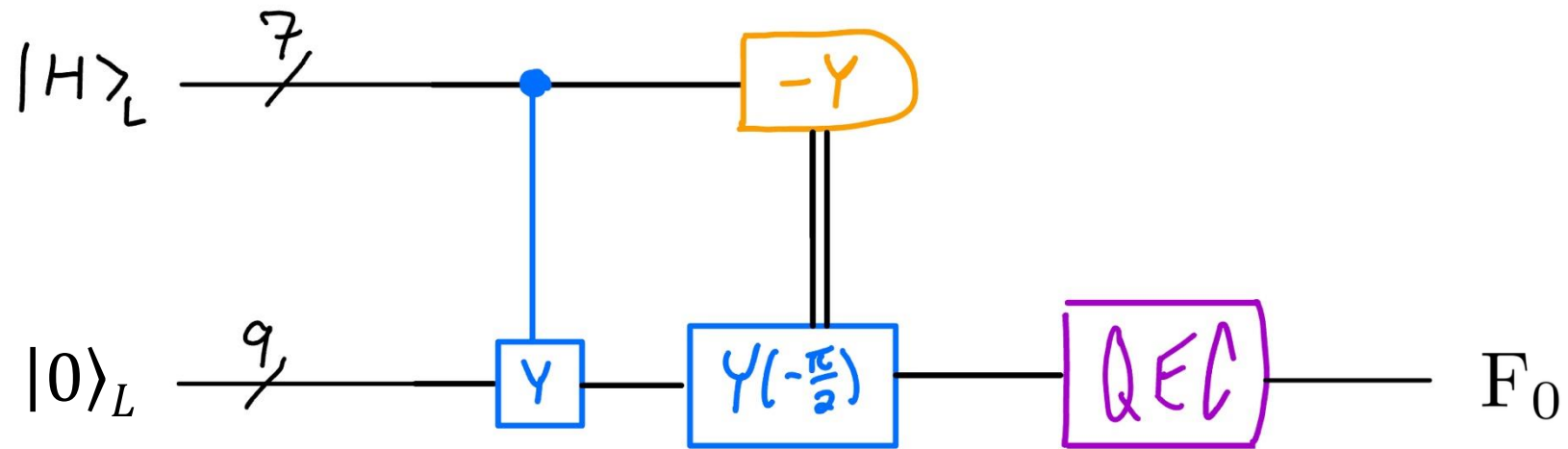
Logical Injection Circuit Tomography



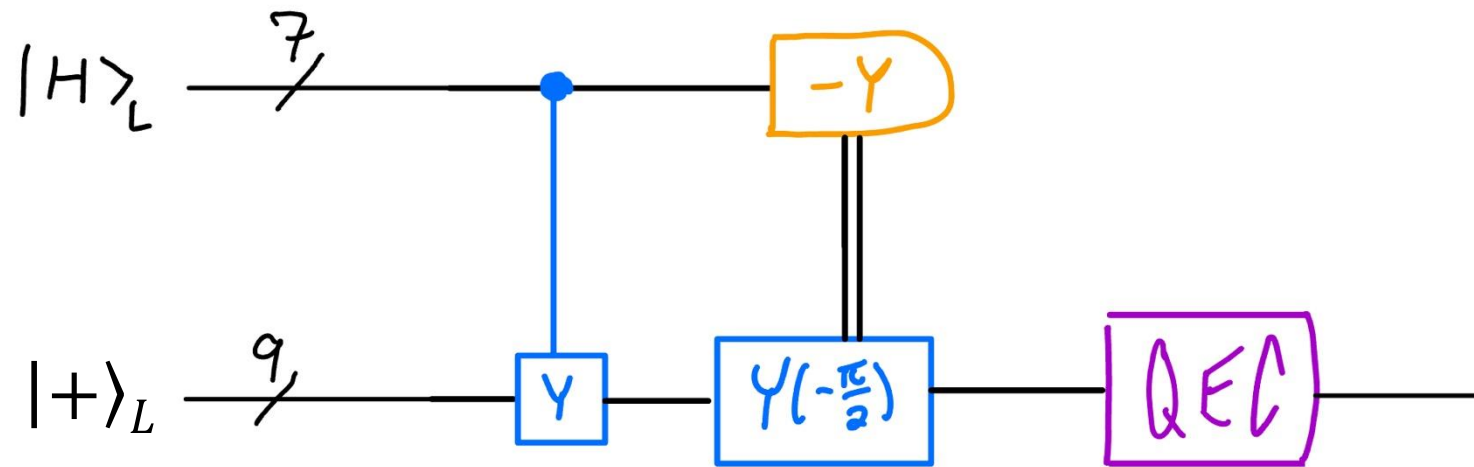
Logical Injection Circuit Tomography



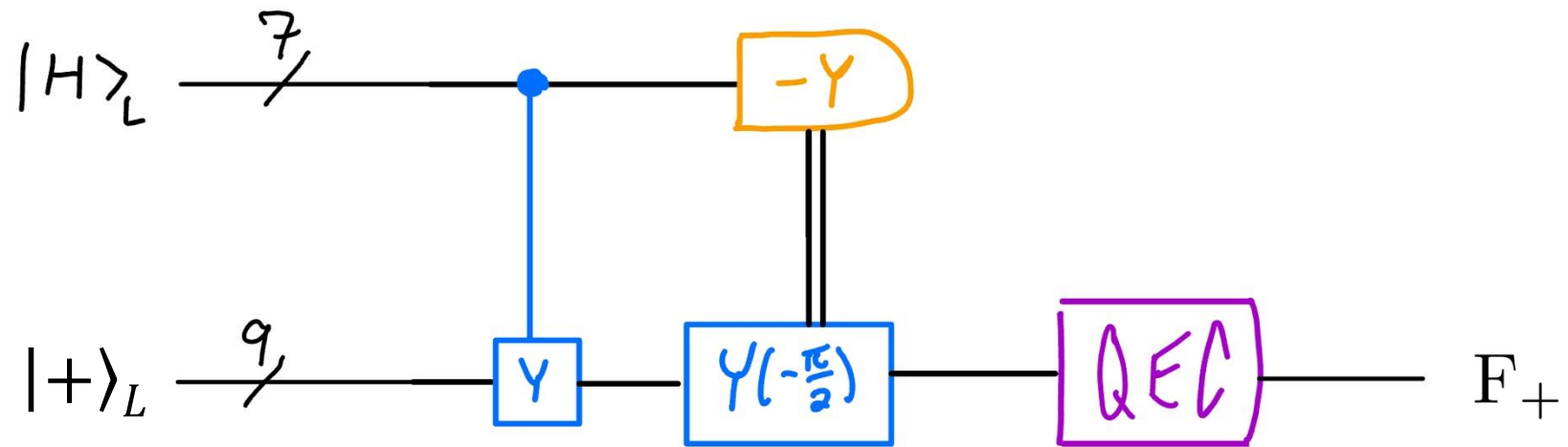
Logical Injection Circuit Tomography



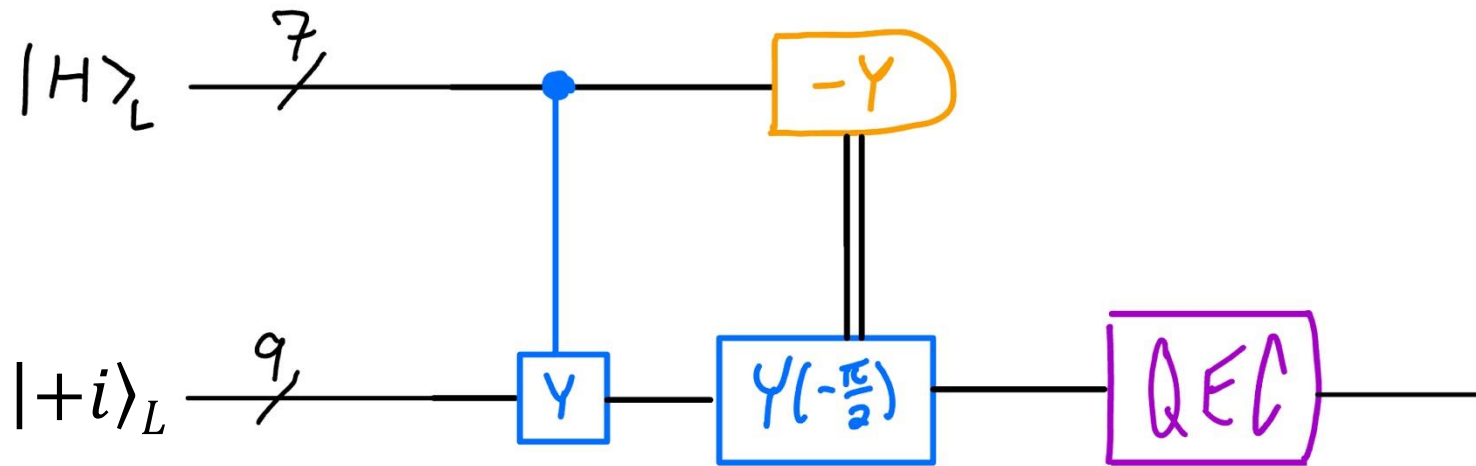
Logical Injection Circuit Tomography



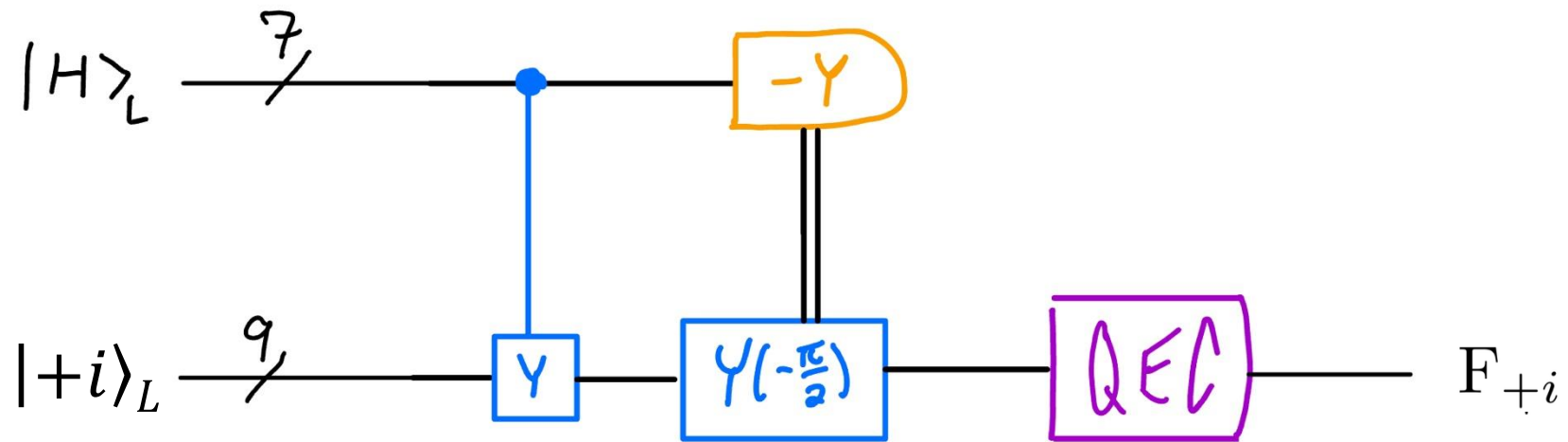
Logical Injection Circuit Tomography



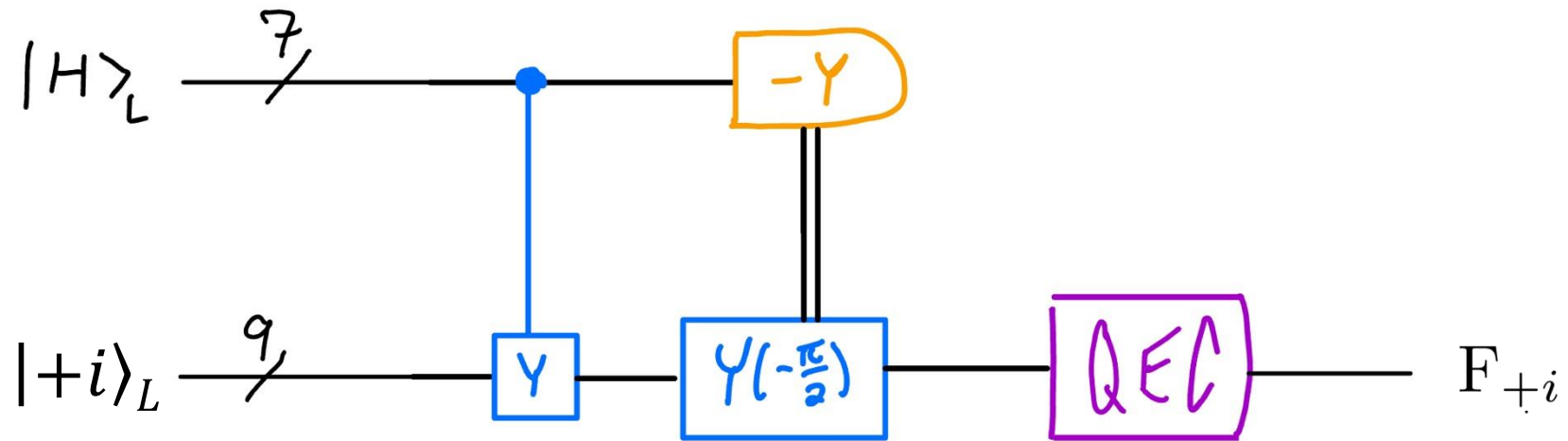
Logical Injection Circuit Tomography



Logical Injection Circuit Tomography



Logical Injection Circuit Tomography



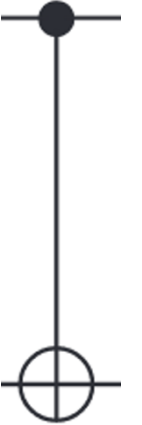
solve system of equations, construct error channel

Two-Qubit Process Tomography (*thanks Aron*)

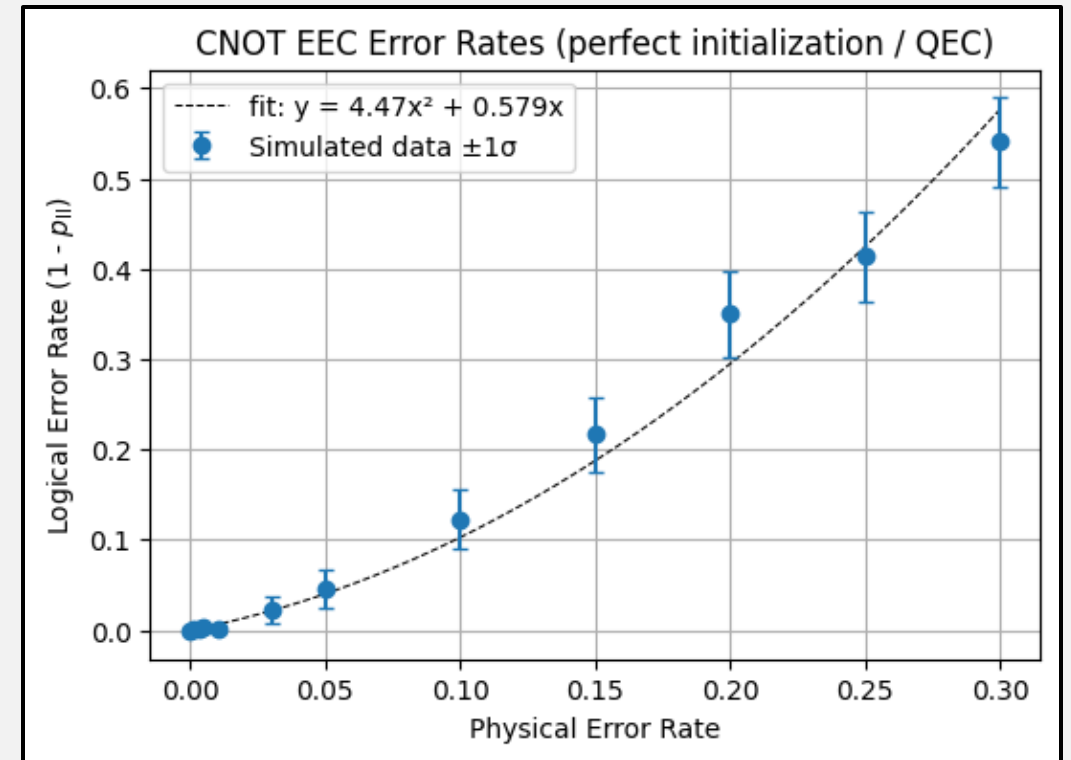
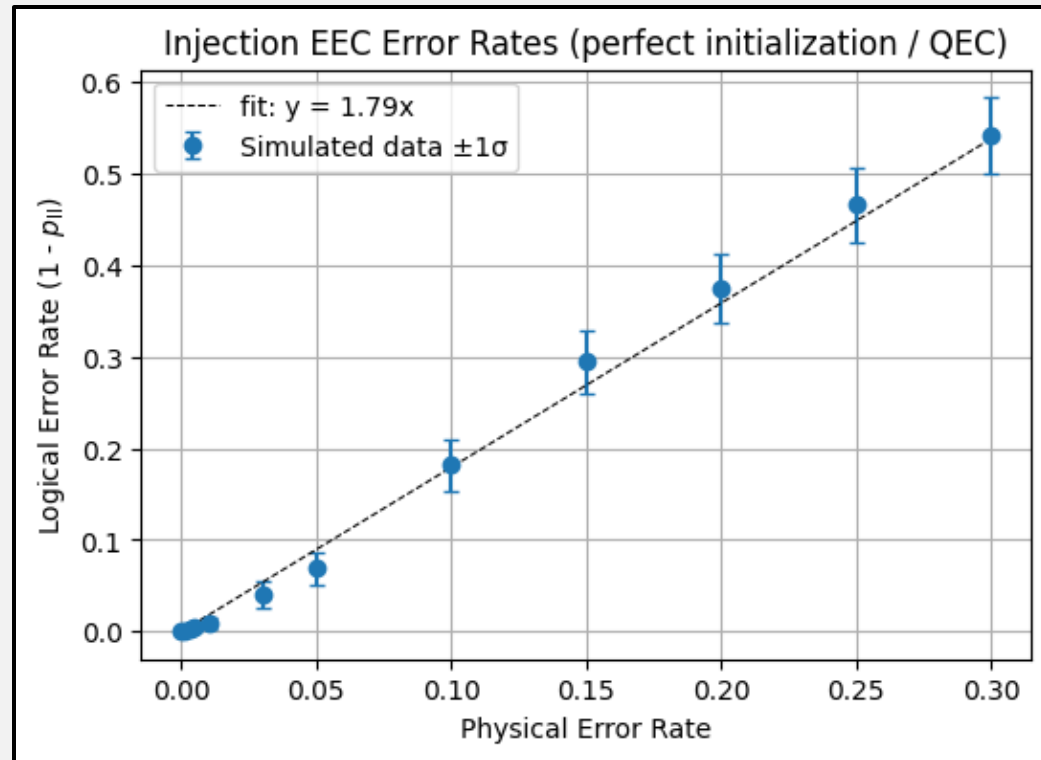
Initial state	Final measurements	Obtained probabilities
$ \overline{00}\rangle$	$\bar{Z}_1, \bar{Z}_2$	$P_{00}^{ZZ}(-,+), P_{00}^{ZZ}(+,-), P_{00}^{ZZ}(-,-)$
$ \overline{0+}\rangle$	$\bar{Z}_1, \bar{X}_2$	$P_{0+}^{ZX}(-,+), P_{0+}^{ZX}(+,-), P_{0+}^{ZX}(-,-)$
$ \overline{0i}\rangle$	$\bar{Z}_1, \bar{Y}_2$	$P_{0i}^{ZY}(-,+), P_{0i}^{ZY}(+,-), P_{0i}^{ZY}(-,-)$
$ \overline{i+}\rangle$	$\bar{Y}_1, \bar{X}_2$	$P_{i+}^{YX}(-,+), P_{i+}^{YX}(+,-), P_{i+}^{YX}(-,-)$
$ \overline{++}\rangle$	$\bar{X}_1, \bar{X}_2$	$P_{++}^{XX}(-,+), P_{++}^{XX}(+,-), P_{++}^{XX}(-,-)$
$ \overline{+0}\rangle$	$\bar{Y}_1 \bar{Y}_2$	$P_{+0}^{YY}(-)$
$ \overline{+i}\rangle$	$\bar{Y}_1 \bar{Z}_2$	$P_{+i}^{YZ}(-)$
$ \overline{i0}\rangle$	$\bar{X}_1 \bar{Y}_2$	$P_{i0}^{XY}(-)$
$ \overline{ii}\rangle$	$\bar{X}_1 \bar{Z}_2$	$P_{ii}^{XZ}(-)$

Two-Qubit Process Tomography (*thanks Aron*)

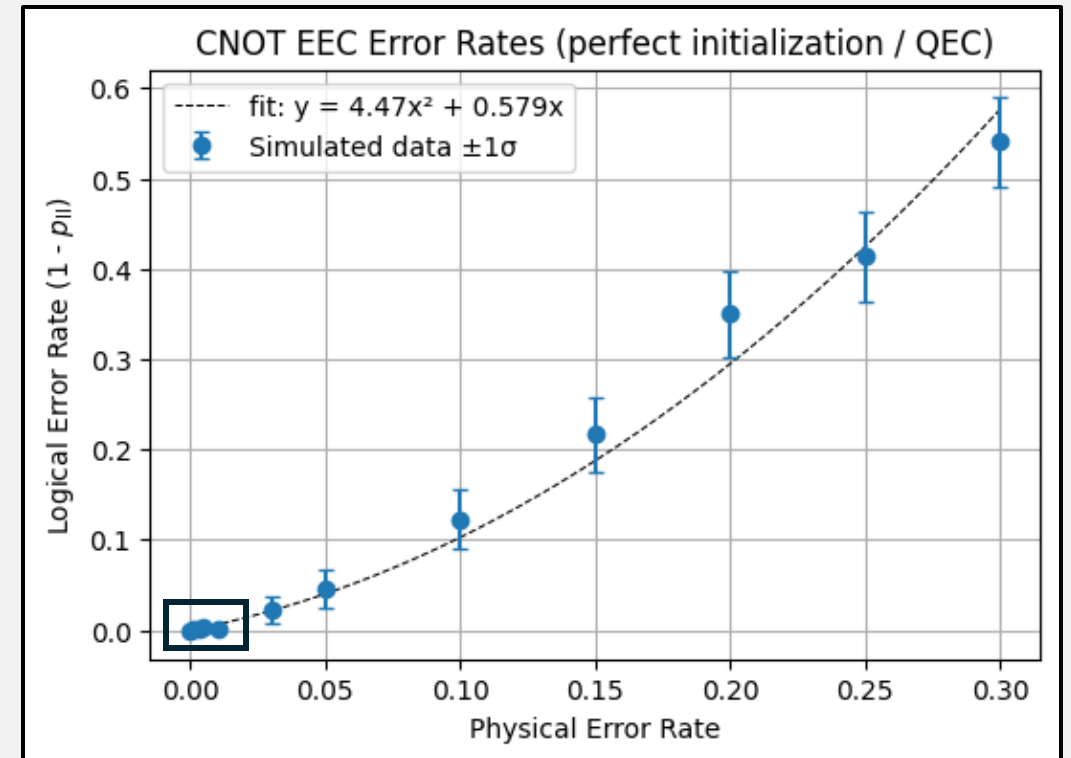
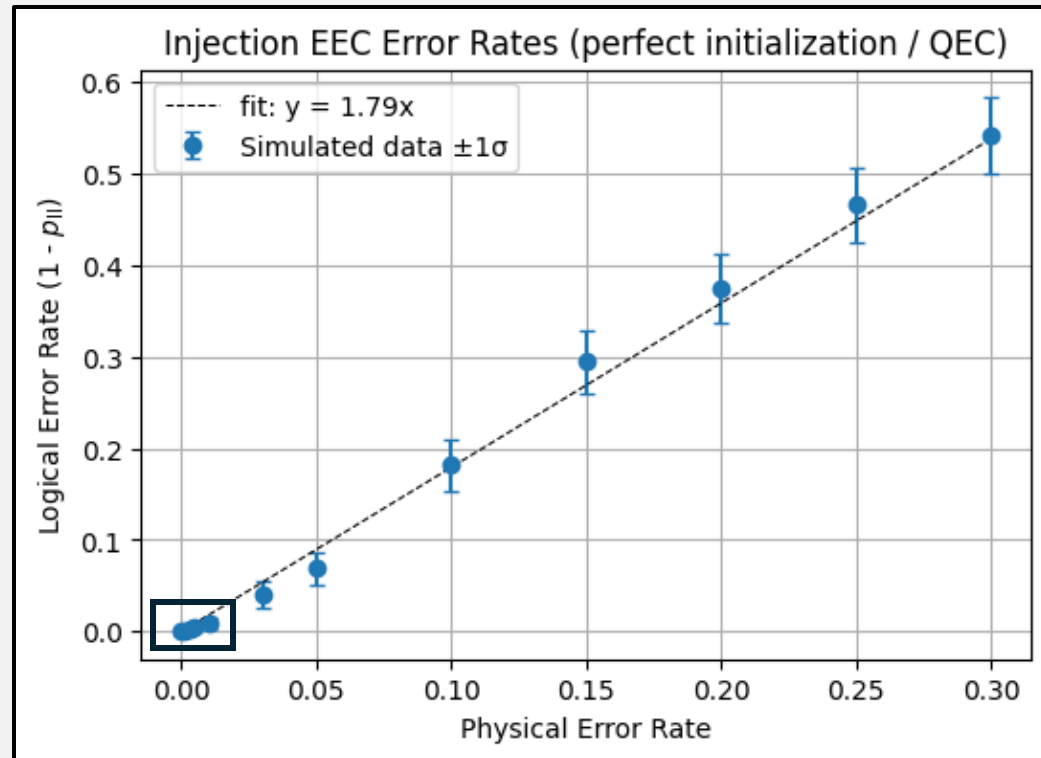
Initial state	Final measurements	Obtained probabilities
$ \overline{00}\rangle$	$\bar{Z}_1, \bar{Z}_2$	$P_{00}^{ZZ}(-,+), P_{00}^{ZZ}(+,-), P_{00}^{ZZ}(-,-)$
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$ \overline{++}\rangle$	$\bar{X}_1, \bar{X}_2$	$P_{++}^{XX}(-,+), P_{++}^{XX}(+,-), P_{++}^{XX}(-,-)$
$ \overline{+0}\rangle$	$\bar{Y}_1 \bar{Y}_2$	$P_{+0}^{YY}(-)$
$ \overline{+i}\rangle$	$\bar{Y}_1 \bar{Z}_2$	$P_{+i}^{YZ}(-)$
$ \overline{i0}\rangle$	$\bar{X}_1 \bar{Y}_2$	$P_{i0}^{XY}(-)$
$ \overline{ii}\rangle$	$\bar{X}_1 \bar{Z}_2$	$P_{ii}^{XZ}(-)$



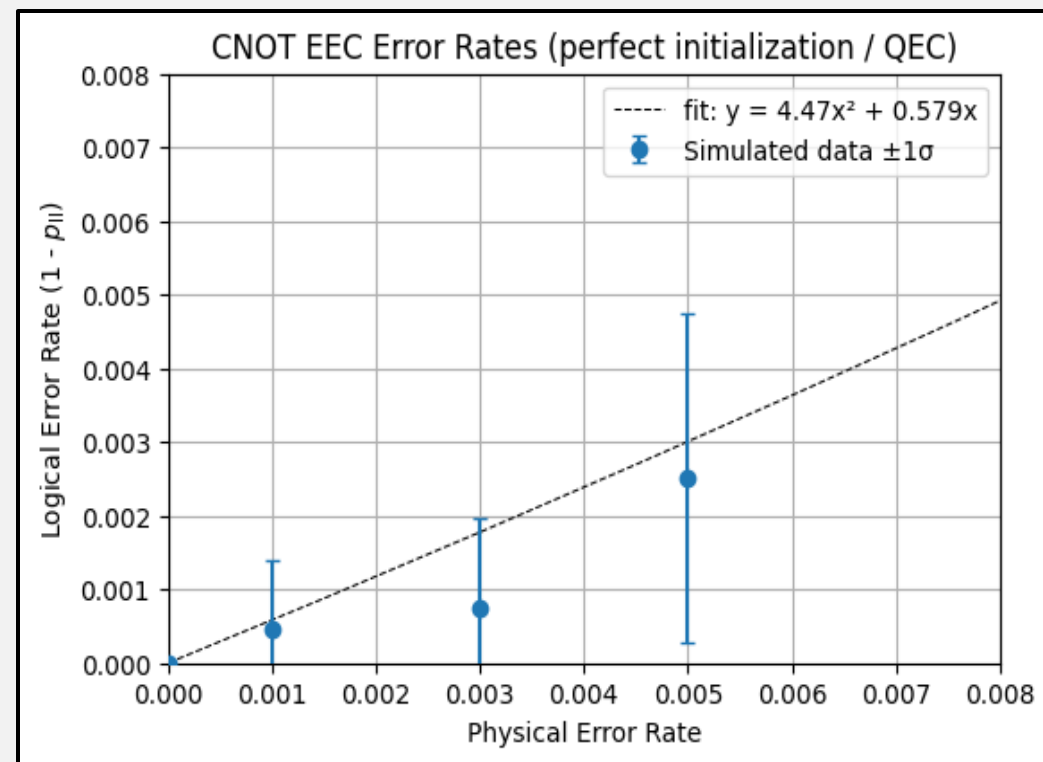
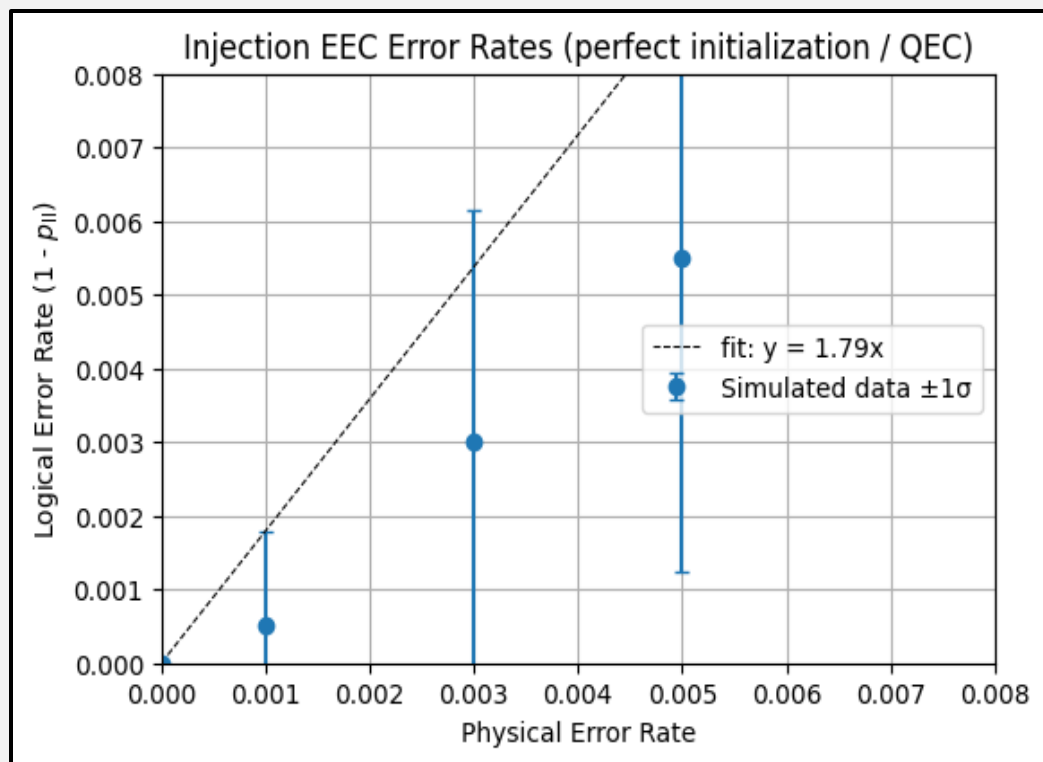
Effective Error Channels PER vs LER



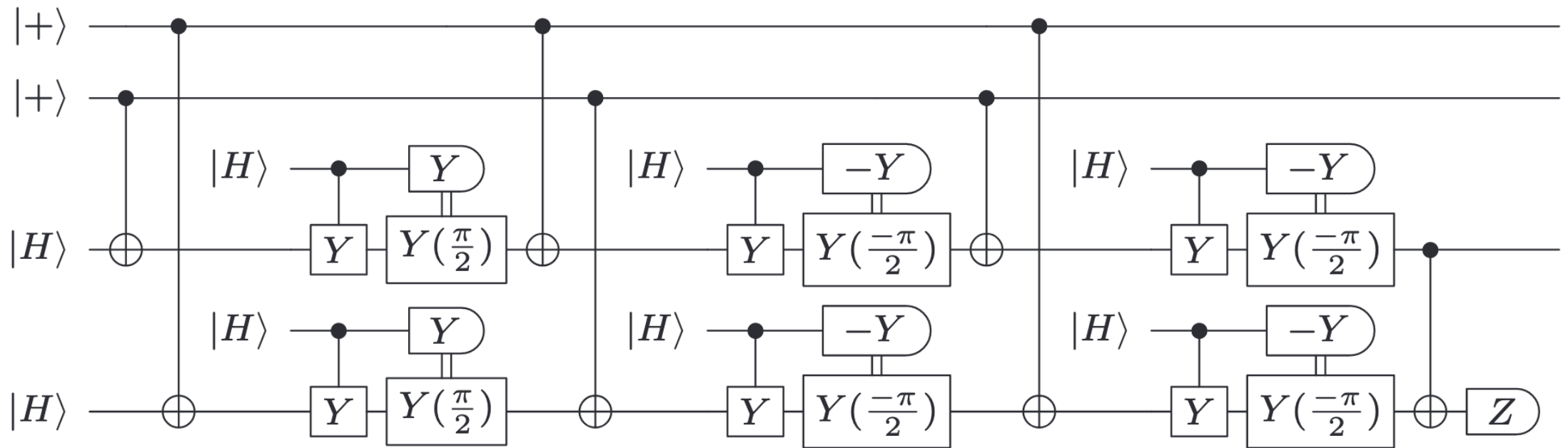
Effective Error Channels PER vs LER



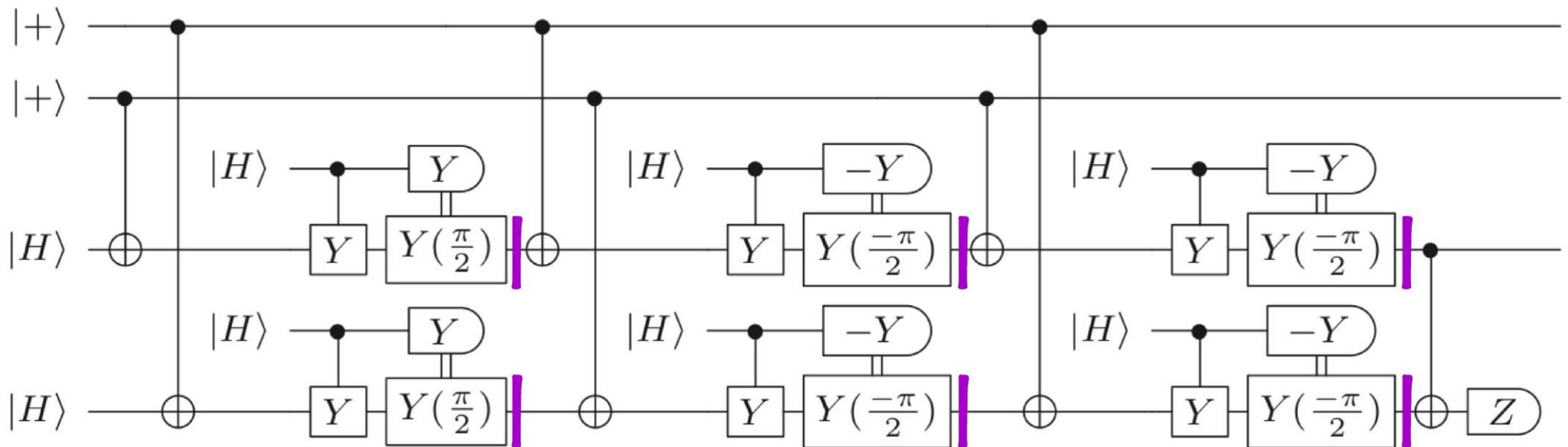
Effective Error Channels PER vs LER



Applying Effective Error Channels to Logical Circuit

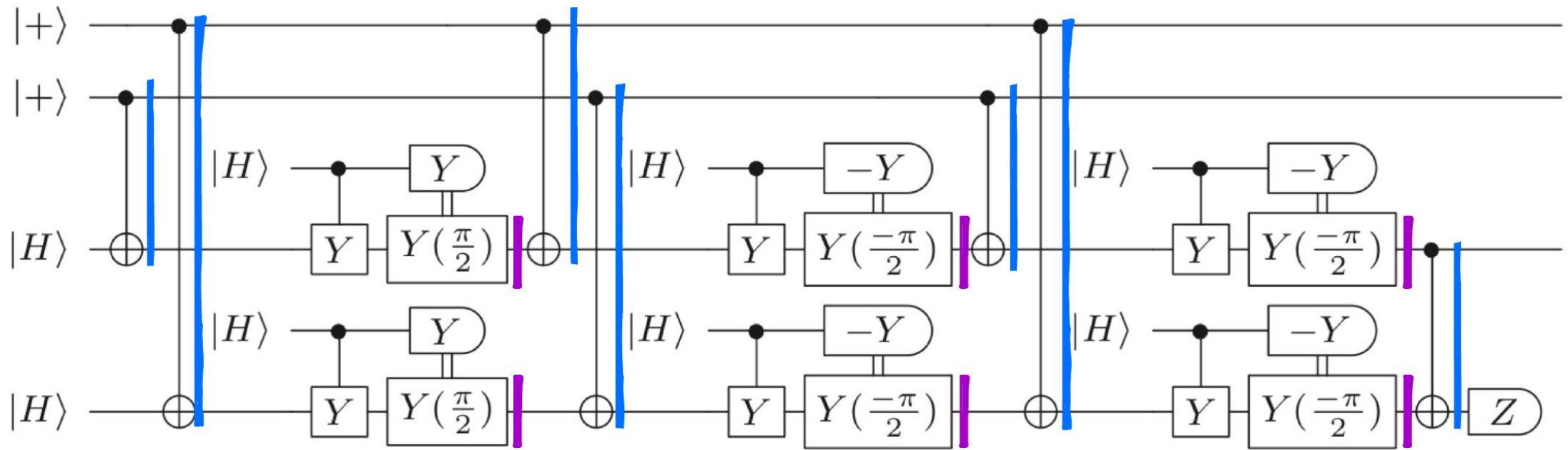


Applying Effective Error Channels to Logical Circuit



= Injection circuit effective error channel

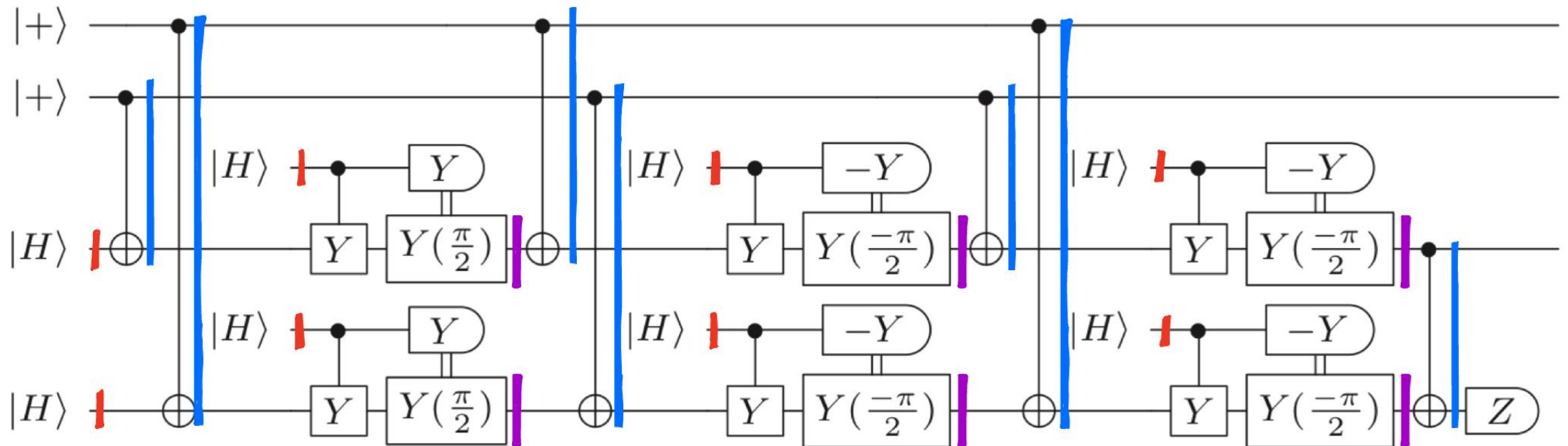
Applying Effective Error Channels to Logical Circuit



 = Injection circuit effective error channel

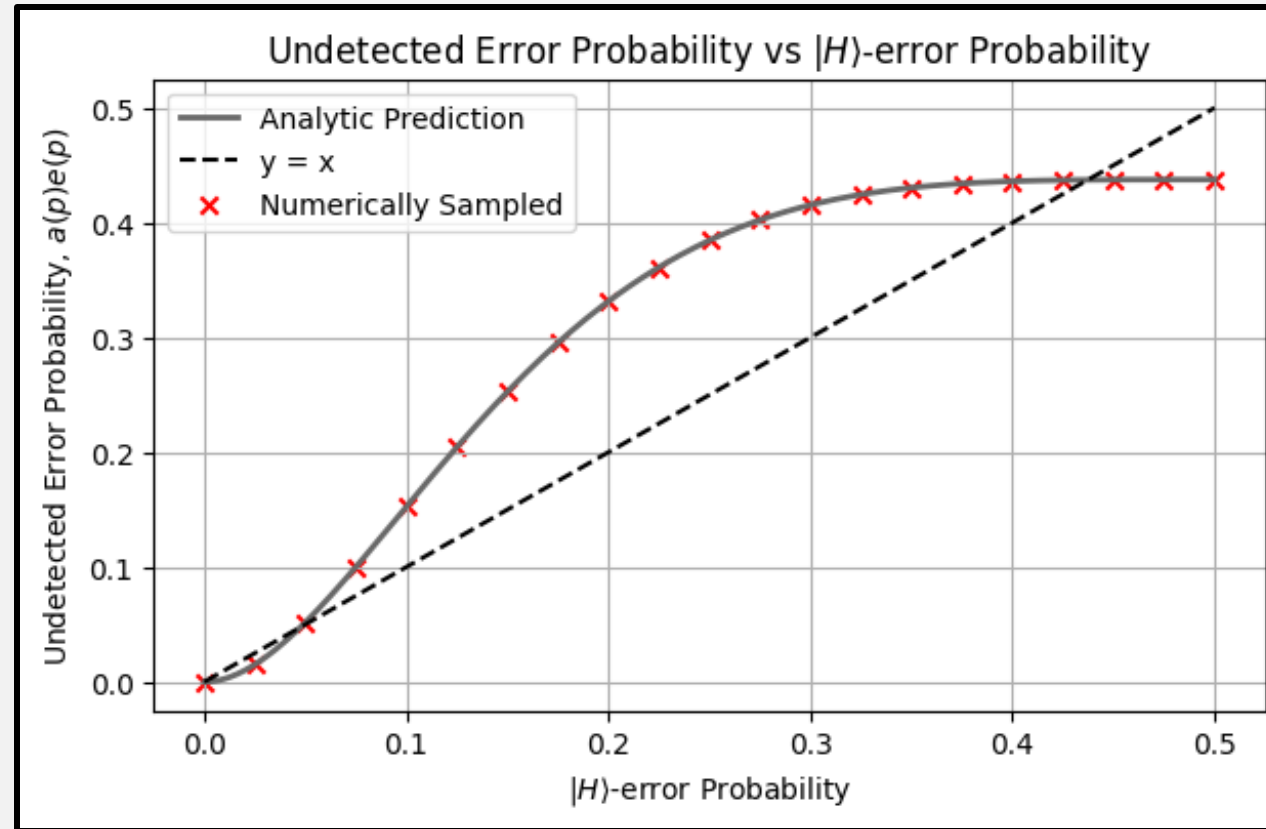
 = CNOT effective error channel

Applying Effective Error Channels to Logical Circuit

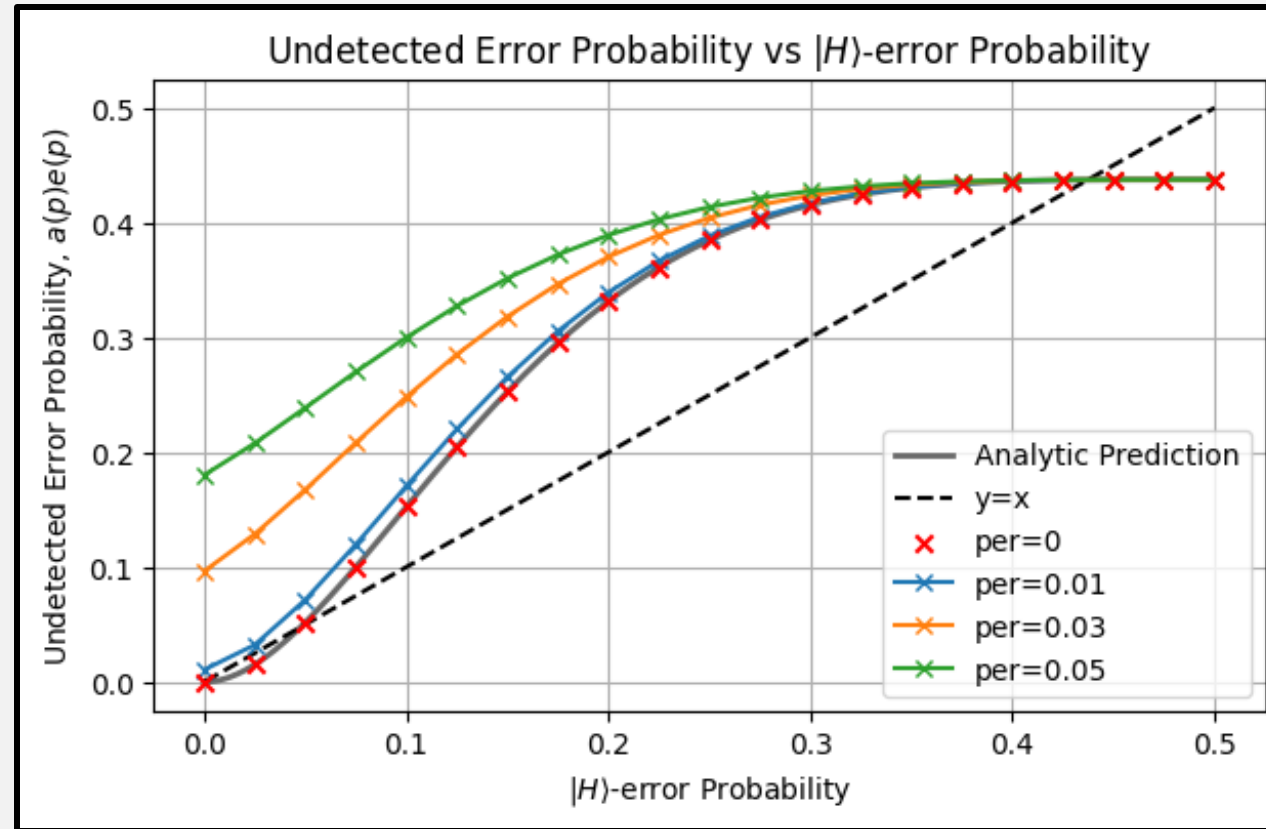


-  = Injection circuit effective error channel
-  = CNOT effective error channel
-  = $|H\rangle$ initialization error channel

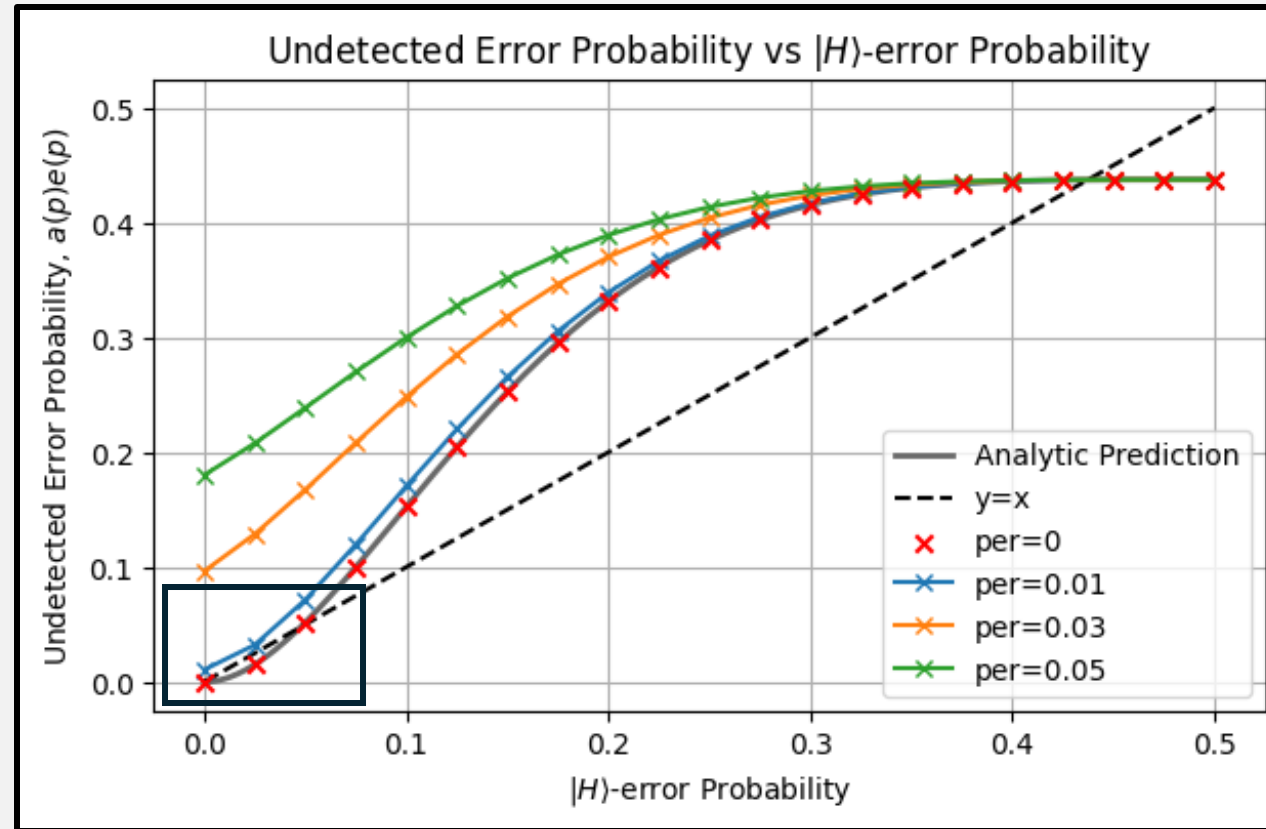
Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



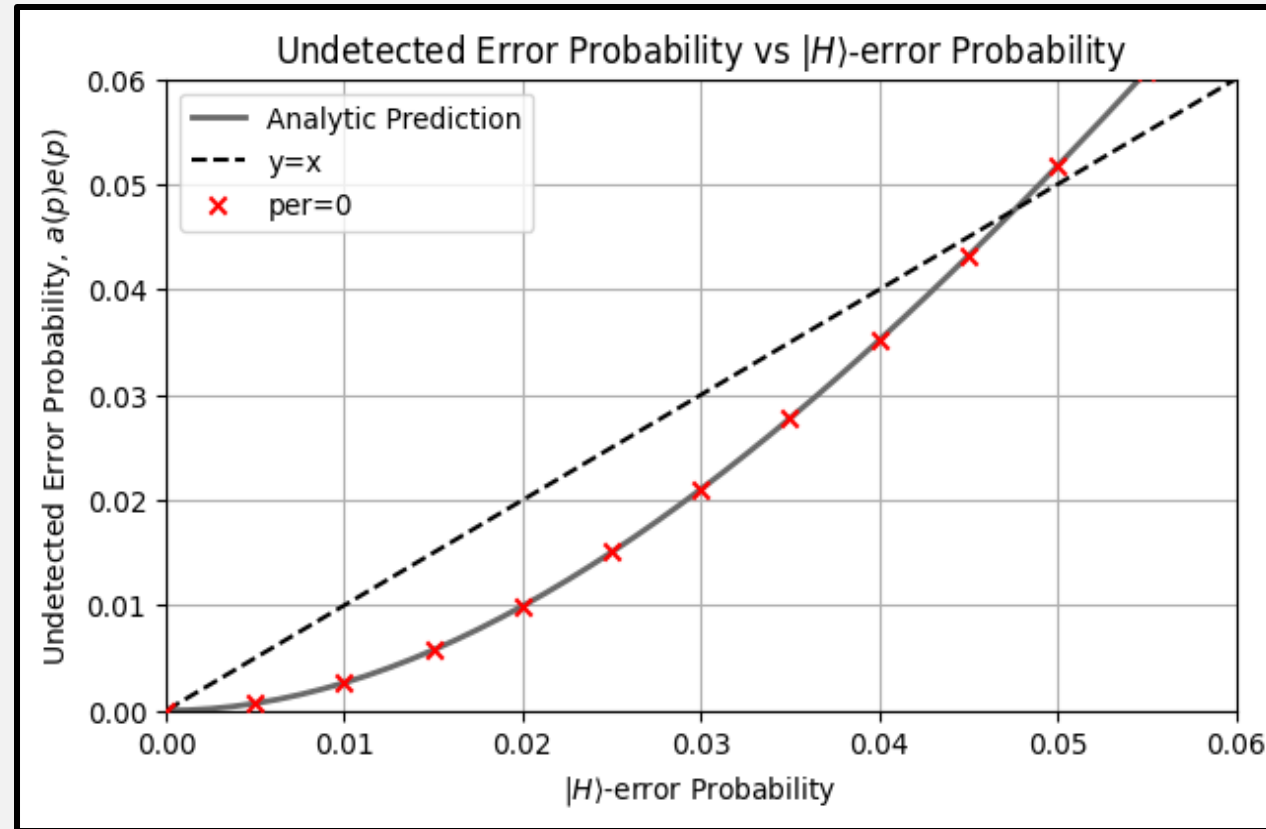
Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



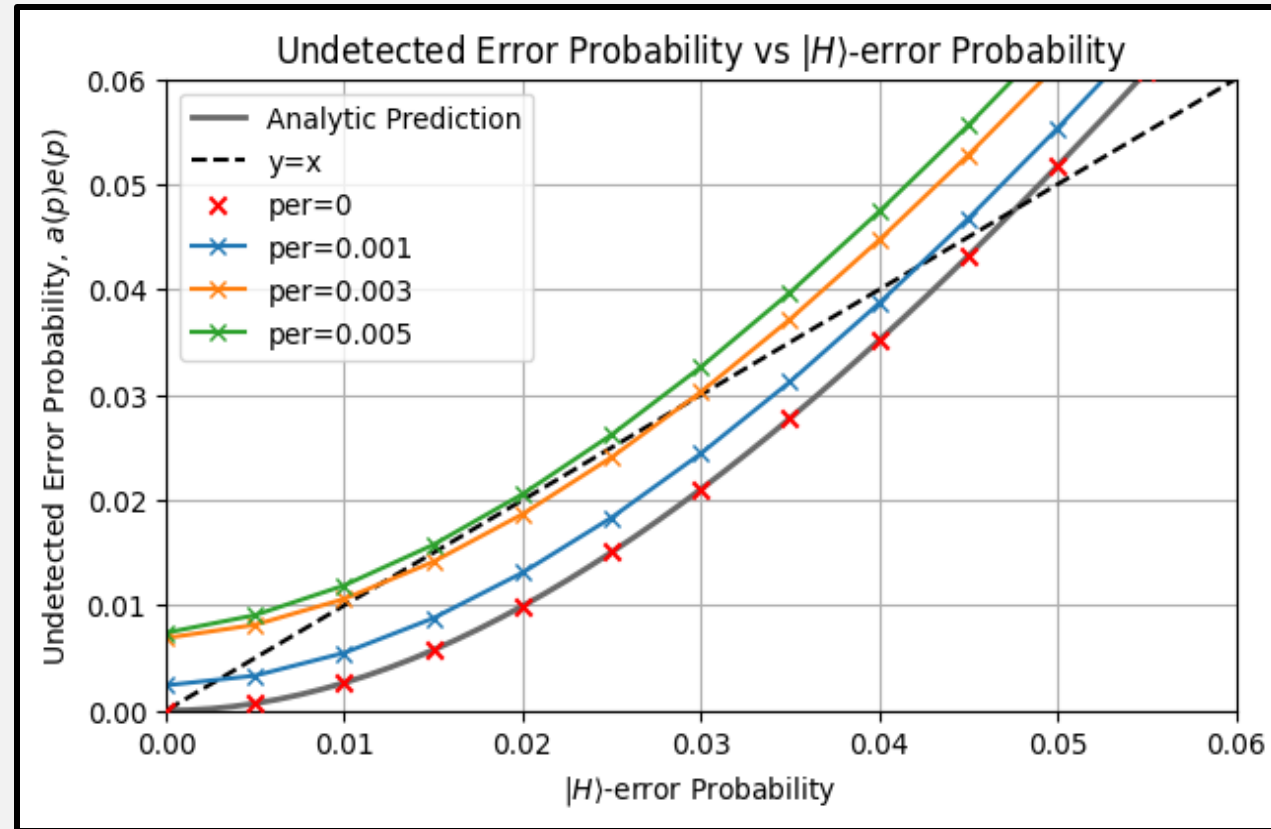
Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



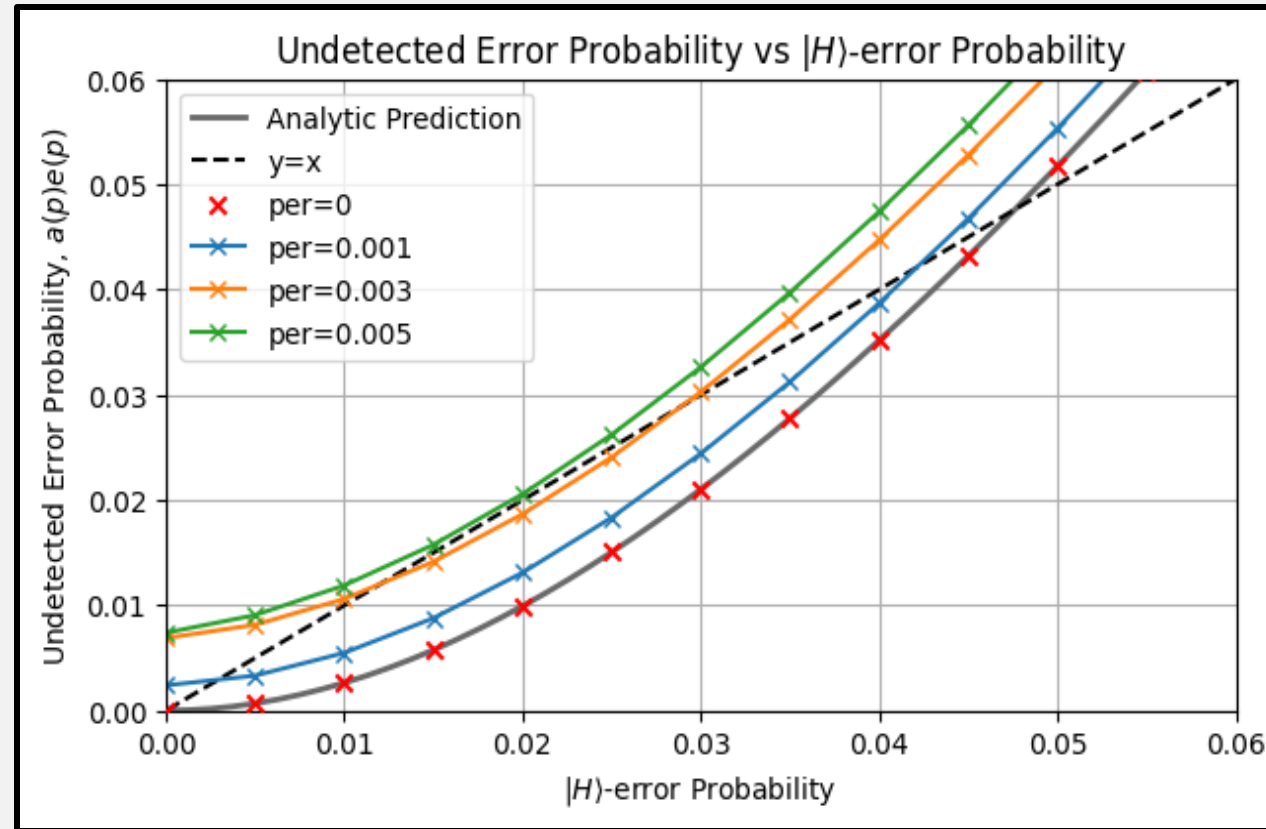
Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



Undetected Error Probability for the Logical Circuit with Circuit-level Noise (Perfect QEC and Initialization)



Need more shots when constructing EECs at small PERs!

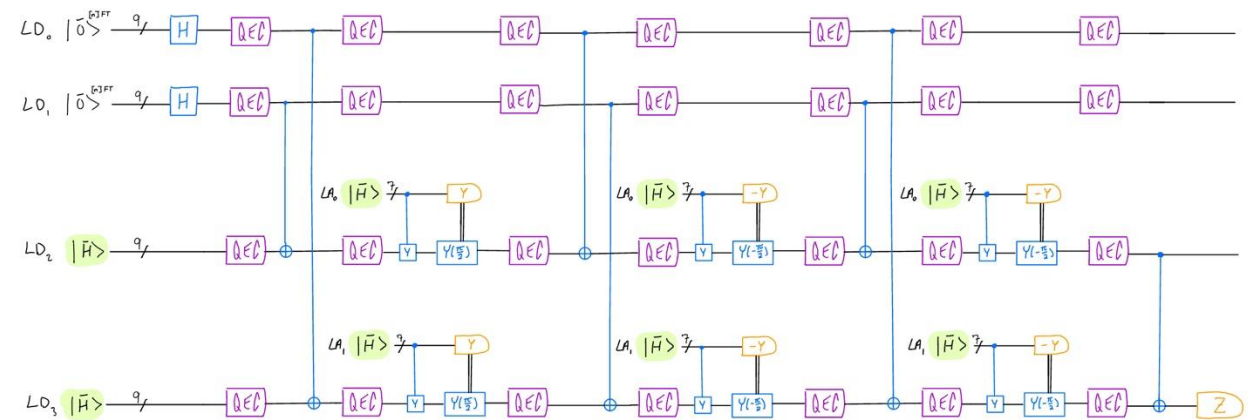
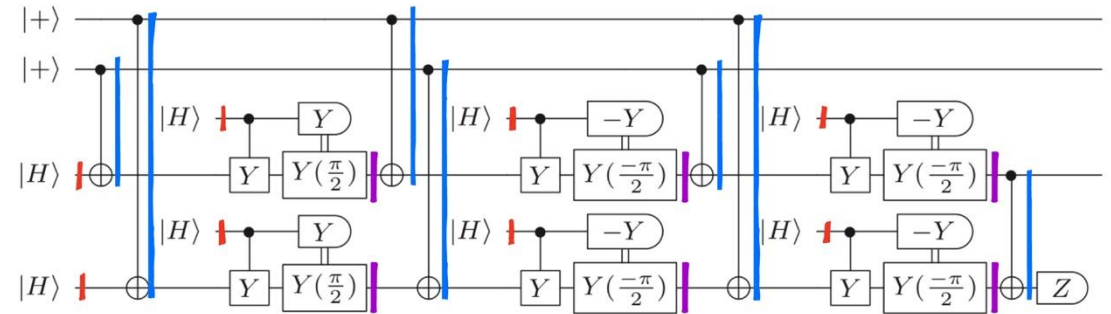
III. Outlook

Simulating Fully-Encoded Circuit for Comparison?

**Simulation of Logical circuit
with effective error channels**

?
=

**Simulation of fully encoded
circuit with Steane code**

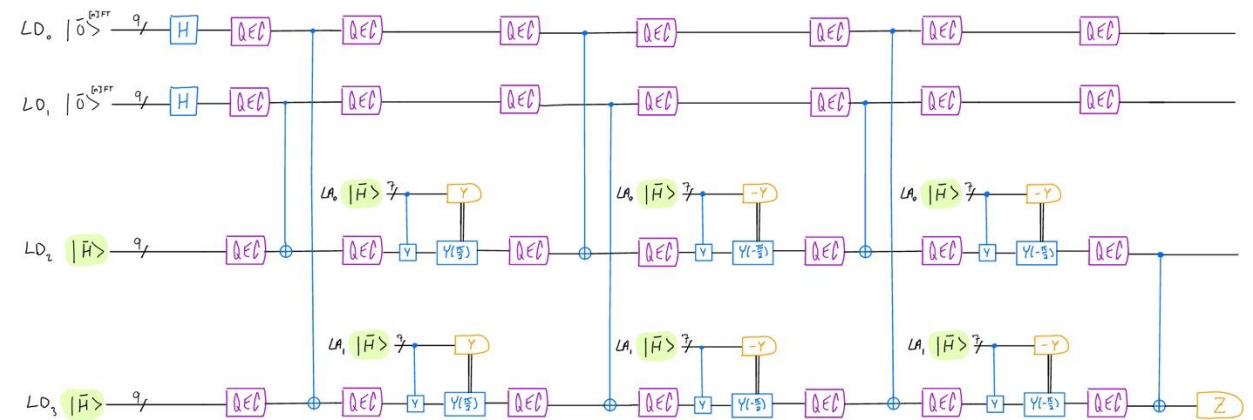
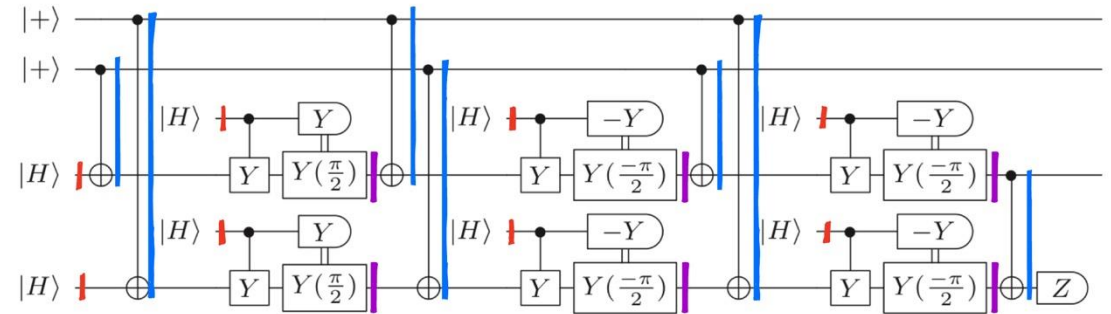


Simulating Fully-Encoded Circuit for Comparison?

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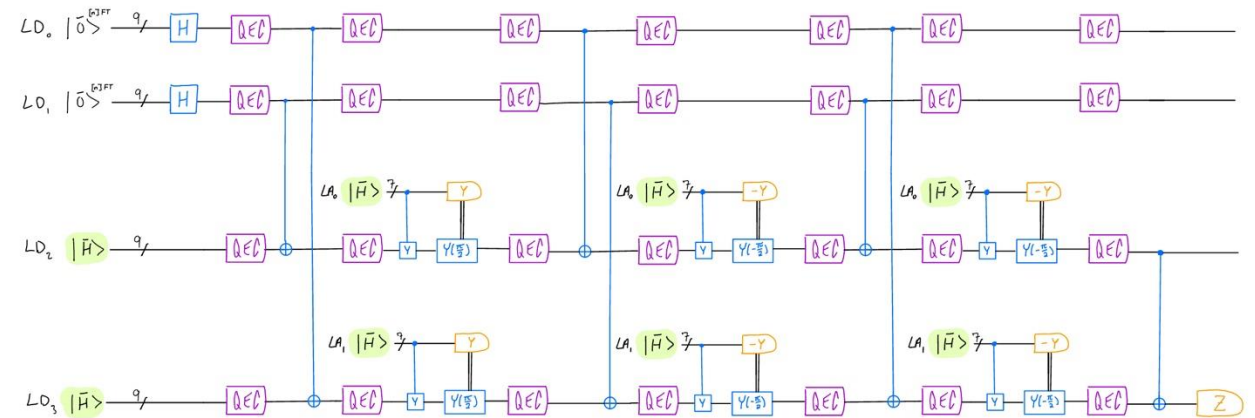
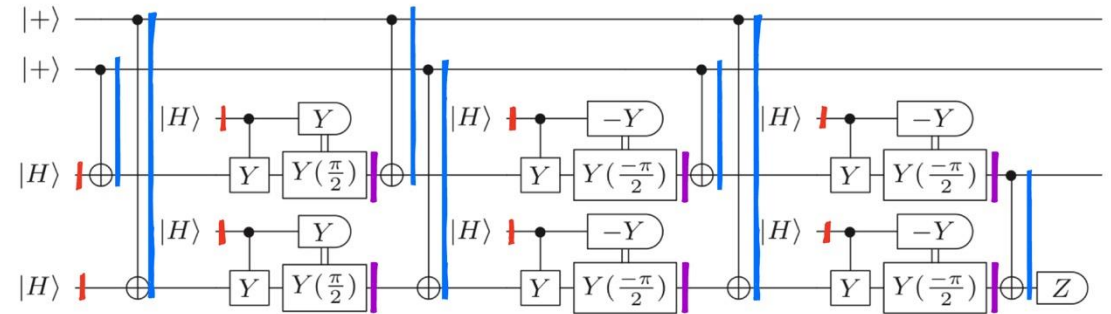
Simulating Fully-Encoded Circuit for Comparison?

Simulation of Logical circuit
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Simulation of fully encoded
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Simulating Fully-Encoded Circuit for Comparison?

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**?
=**

**Simulation of fully encoded
circuit with Steane code**

?

Qiskit's "extended_stabilizer"
simulator

- Computation scales with the number of non-Cliffords

Simulating Fully-Encoded Circuit for Comparison?

**Simulation of Logical circuit
with effective error channels**



**?
=**

**Simulation of fully encoded
circuit with Steane code**

?

Qiskit's "extended_stabilizer"
simulator

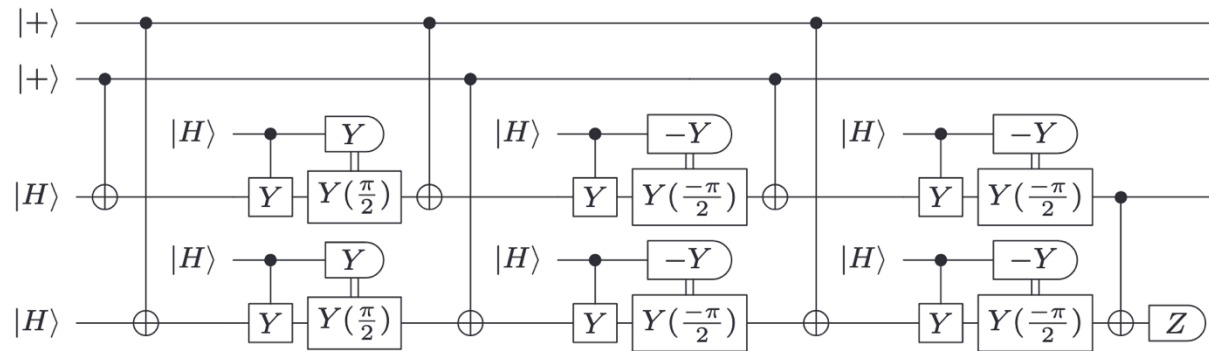
- Computation scales with the number of non-Cliffords

Problem:

- **The simulator does not support classically-conditioned operations mid-circuit**

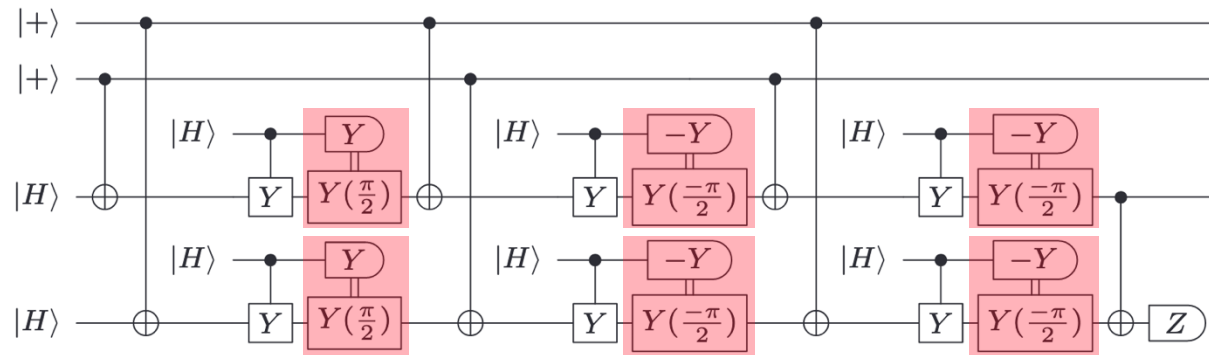
Simulating Fully-Encoded Circuit for Comparison?

Can we track the necessary recovery operations through the circuit?



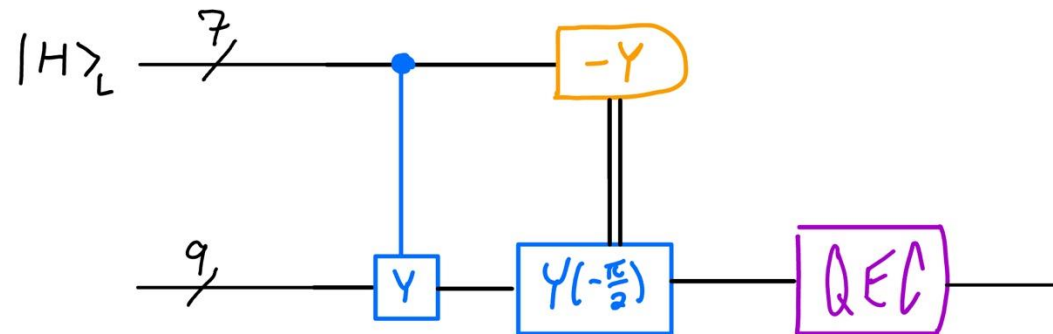
Simulating Fully-Encoded Circuit for Comparison?

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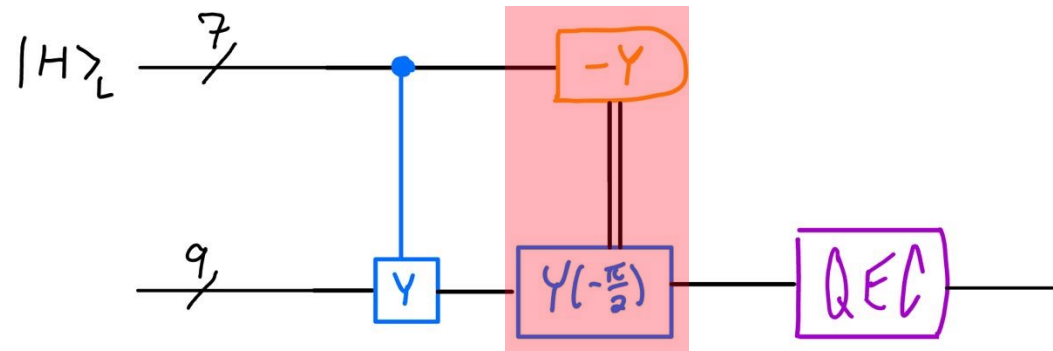
Simulating Fully-Encoded Circuit for Comparison?

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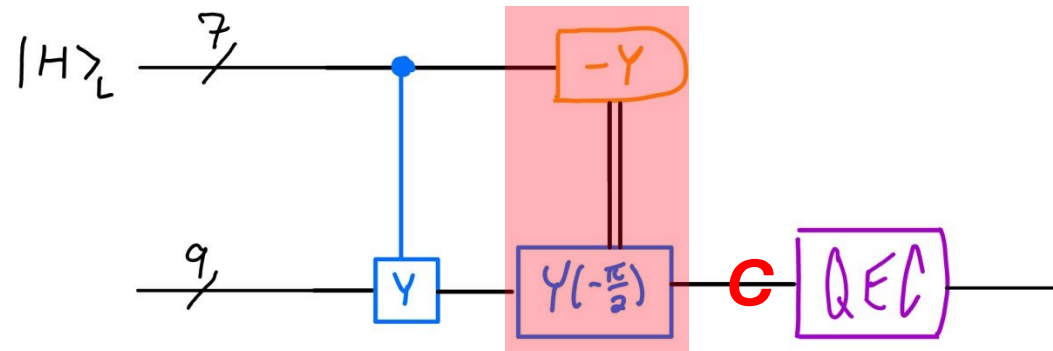
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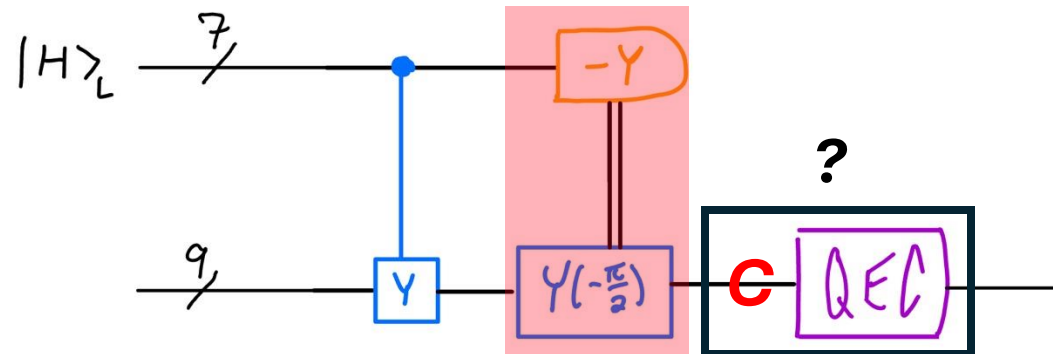
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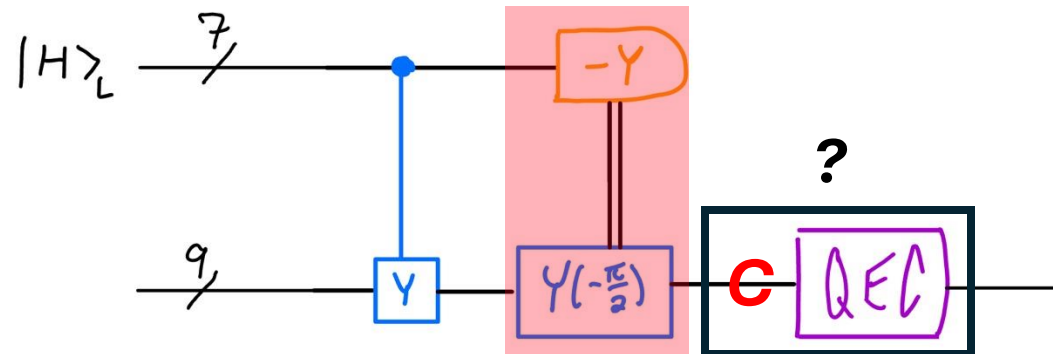
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Simulating Fully-Encoded Circuit for Comparison?

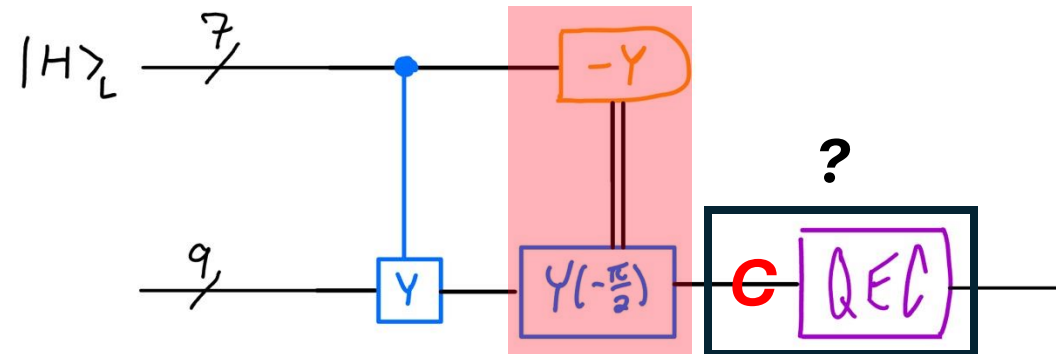
Can we track the necessary recovery operations through the circuit?



Other things:

Simulating Fully-Encoded Circuit for Comparison?

Can we track the necessary recovery operations through the circuit?

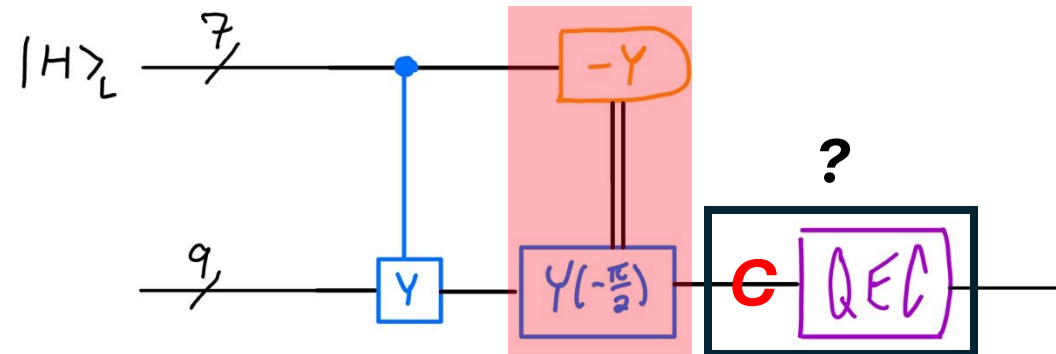


Other things:

somehow modify the injection circuit so that it only contains a Pauli correction?

Simulating Fully-Encoded Circuit for Comparison?

Can we track the necessary recovery operations through the circuit?



Other things:

somehow modify the injection circuit so that it only contains a Pauli correction?

somehow use the algorithmic fault tolerance discussed by Luis in journal club?

Final Outlook

- Optimize code, take more shots when defining EECs for small PER

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 - Or limit scope of question? Just simulate injection circuit for comparison?

Questions ?

Extra Slides

Encoding Logical Qubits

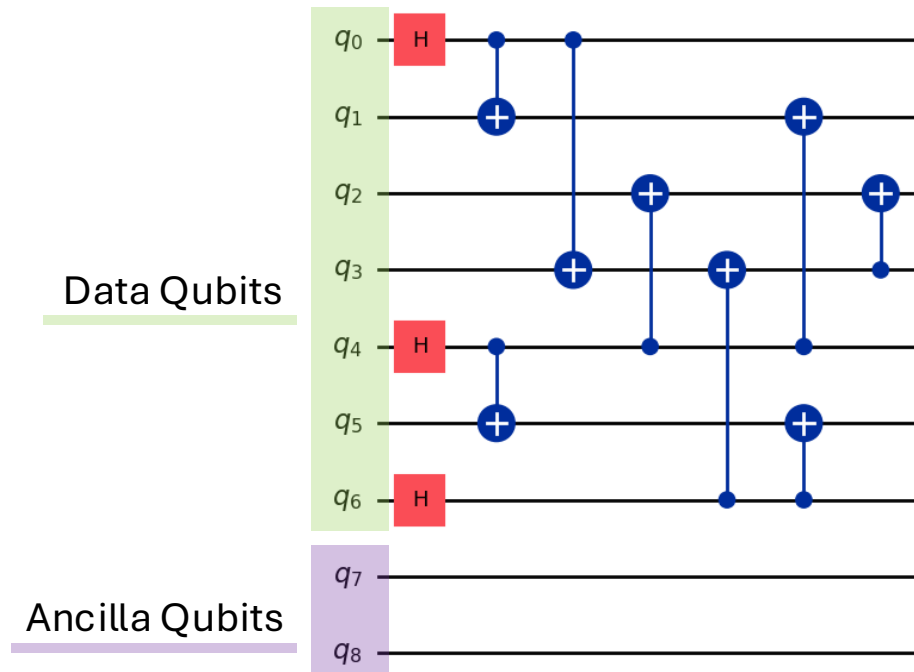
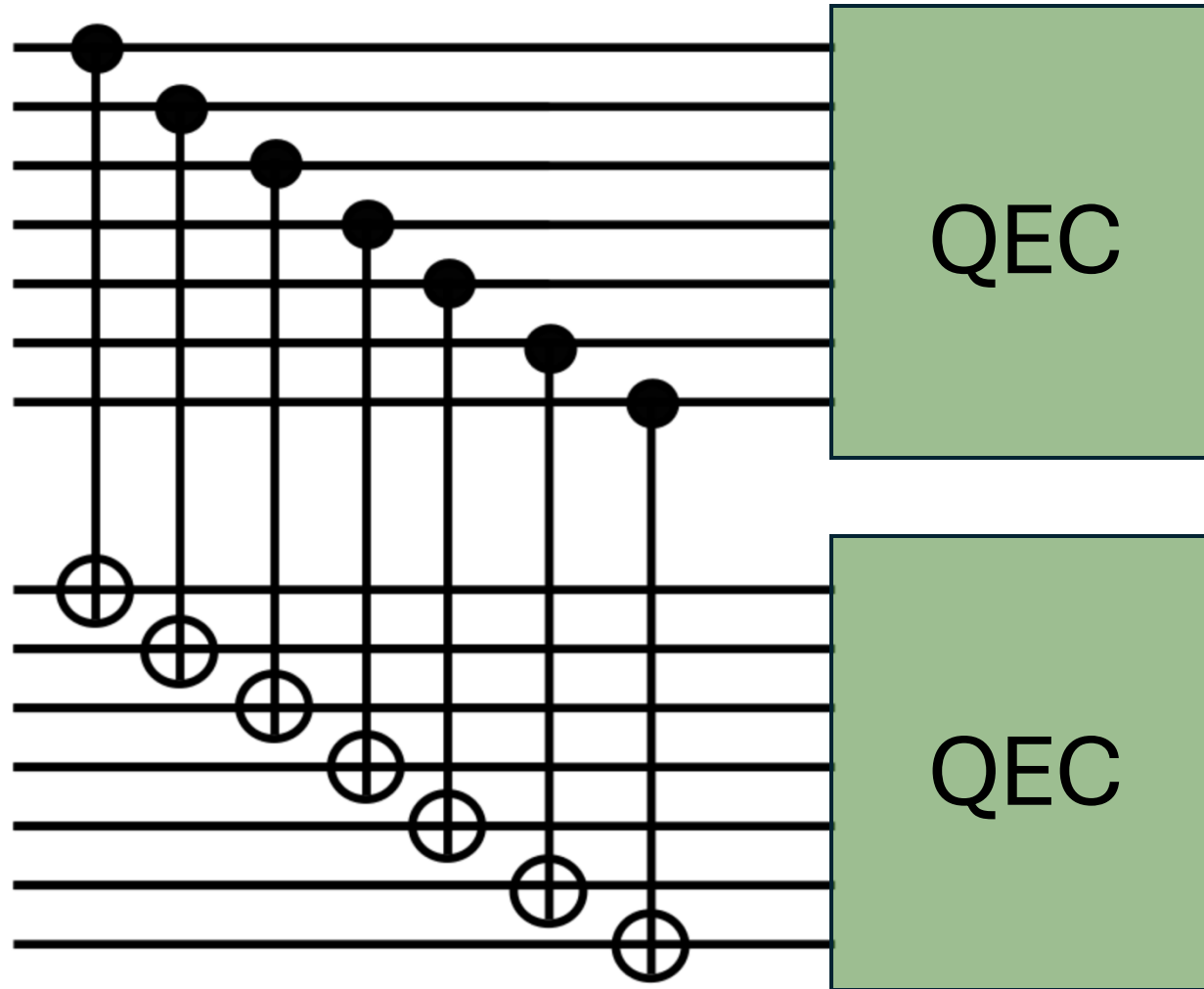


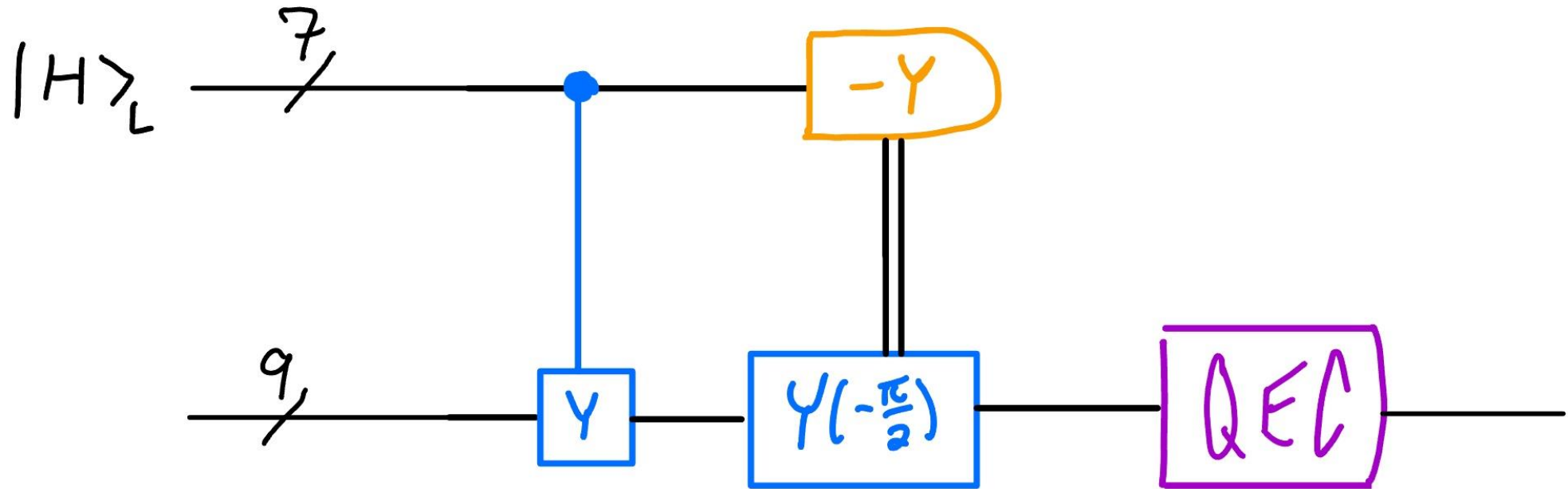
FIG. 6. Encoding circuit for the $|0\rangle_L$ of the Steane code using an initial state of $|0\rangle^{\otimes 7}$.

Intuition for p^2 CNOT LER scaling

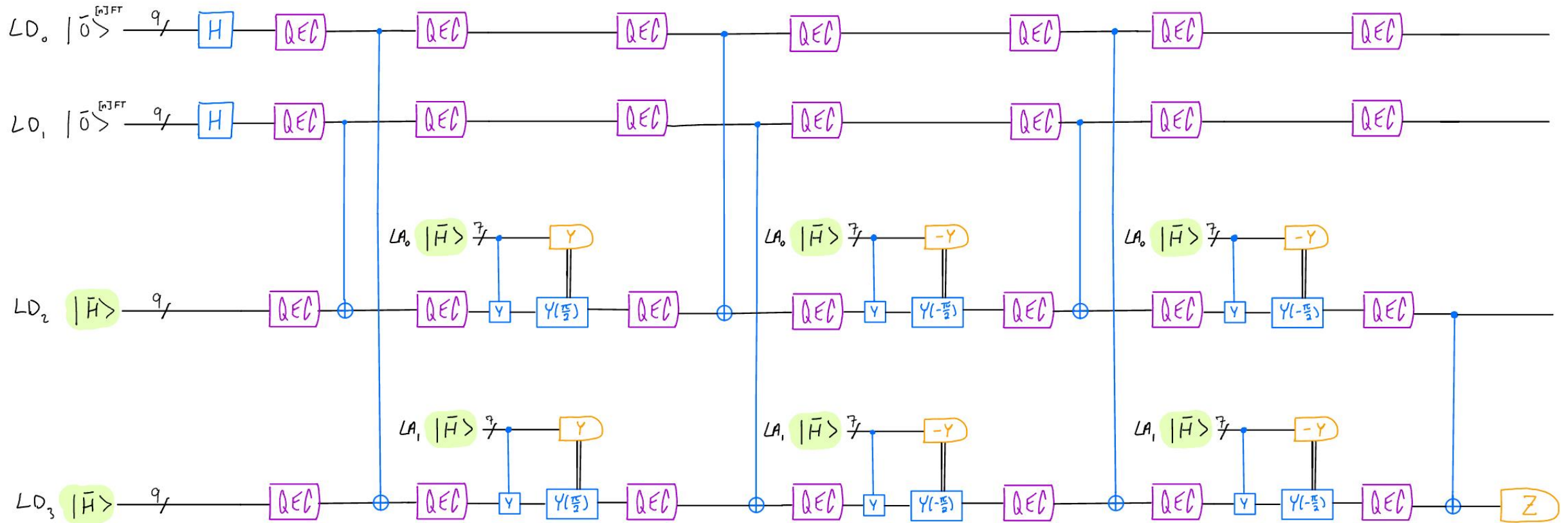


Intuition for Linear Injection LER scaling (?)

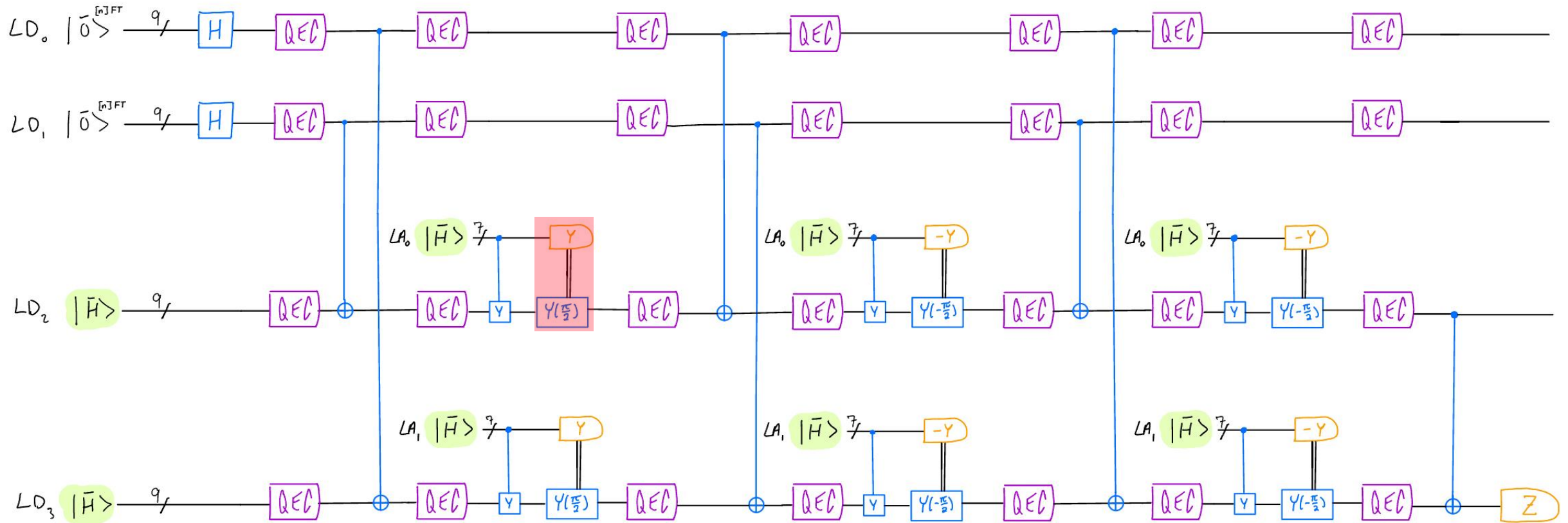
(to leading order)



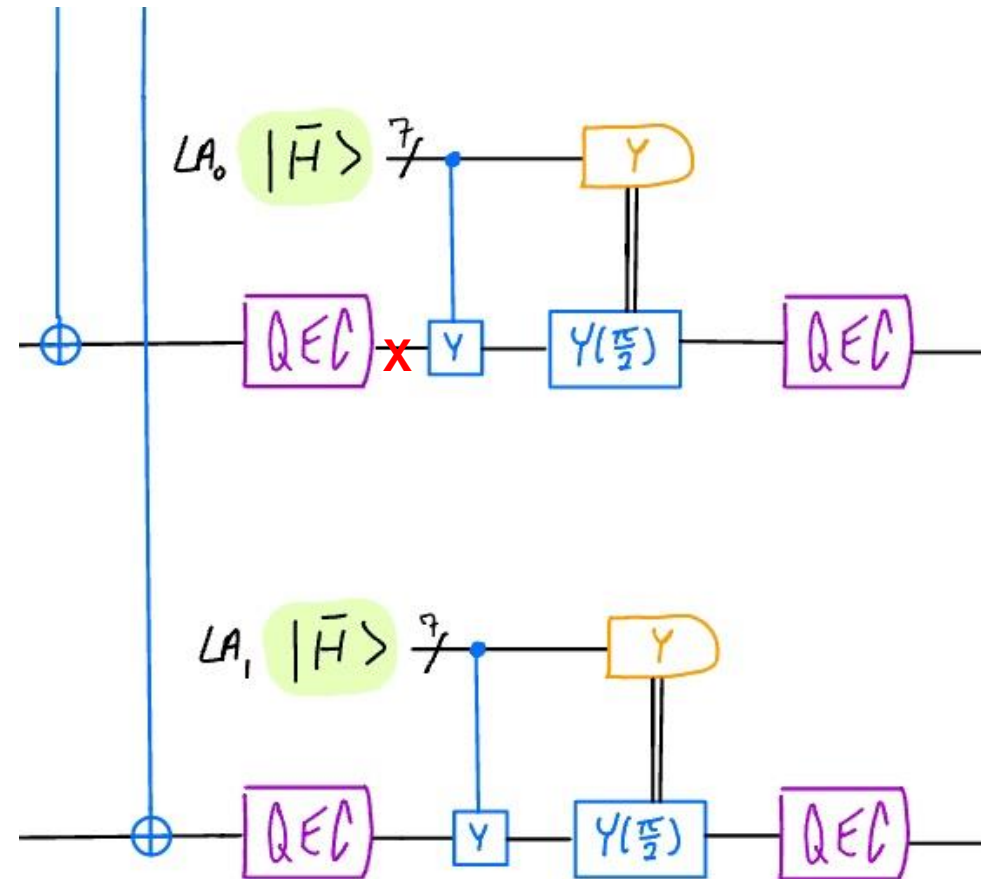
Tracking the "Injection Correction": Possible or not?



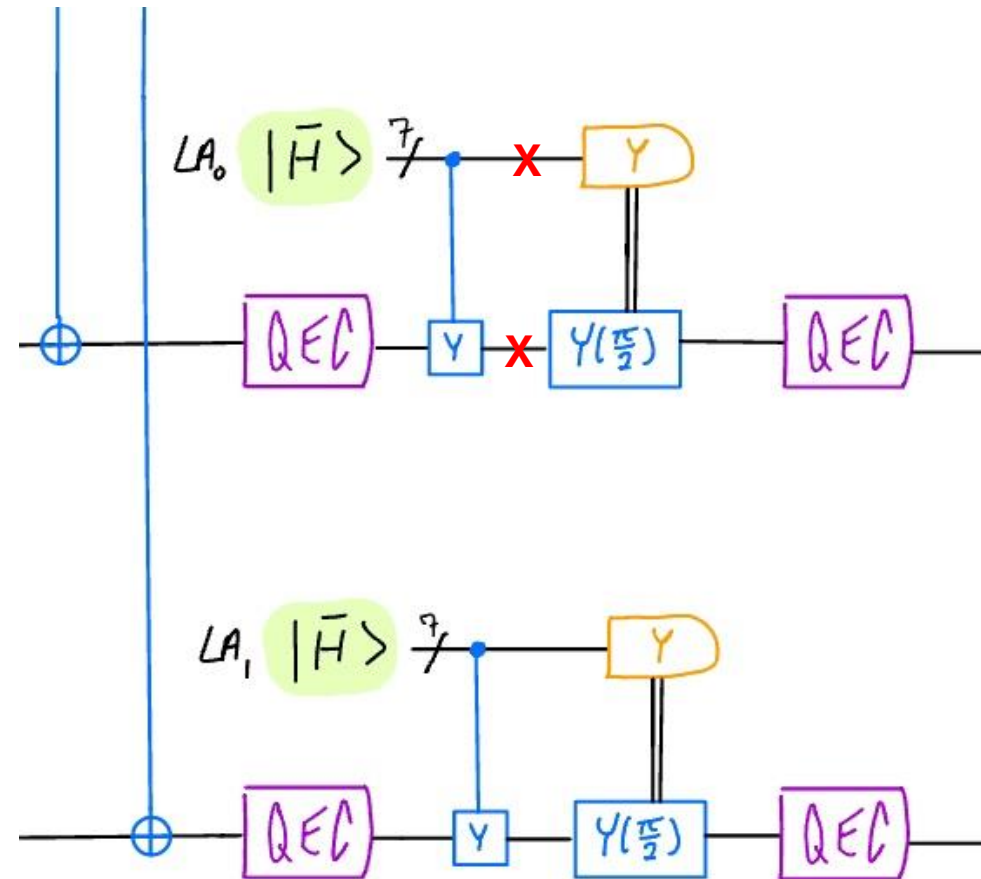
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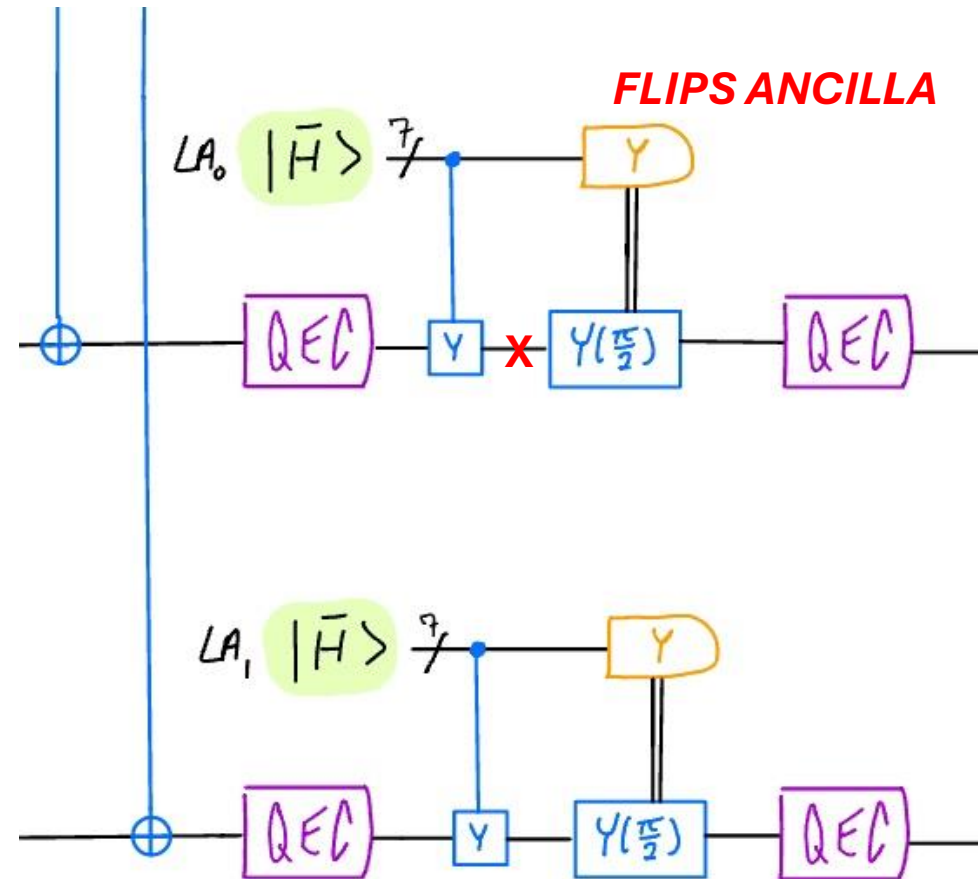
Extra: Notice that the Ancilla Measurement is Unreliable



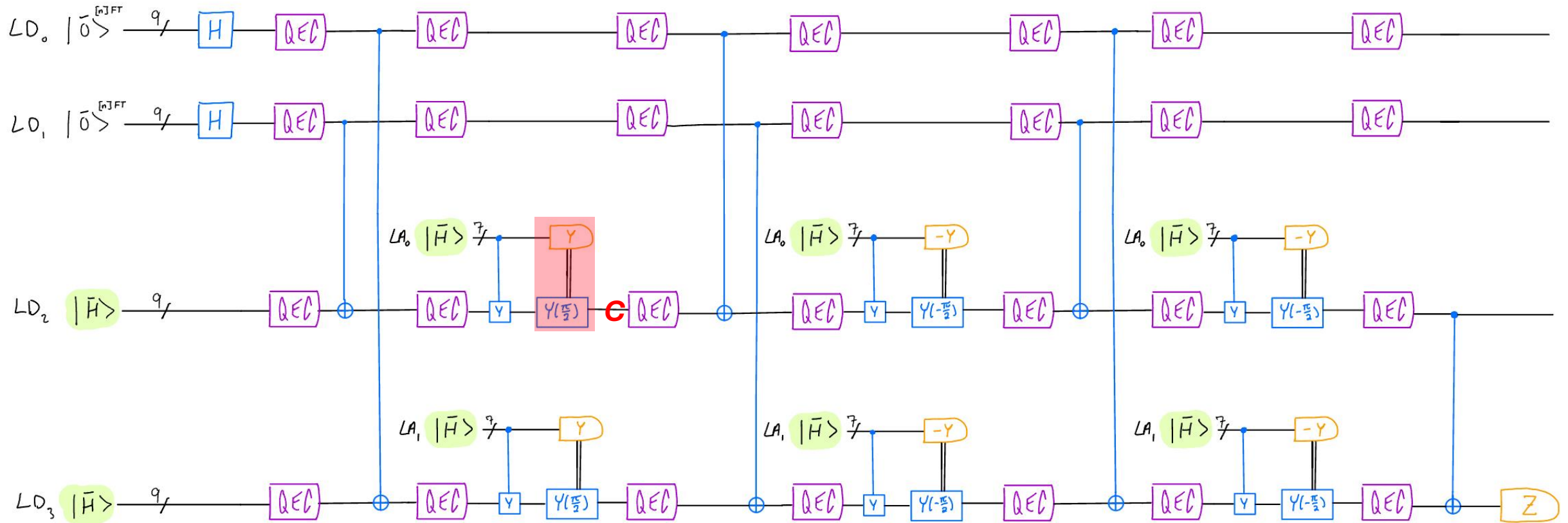
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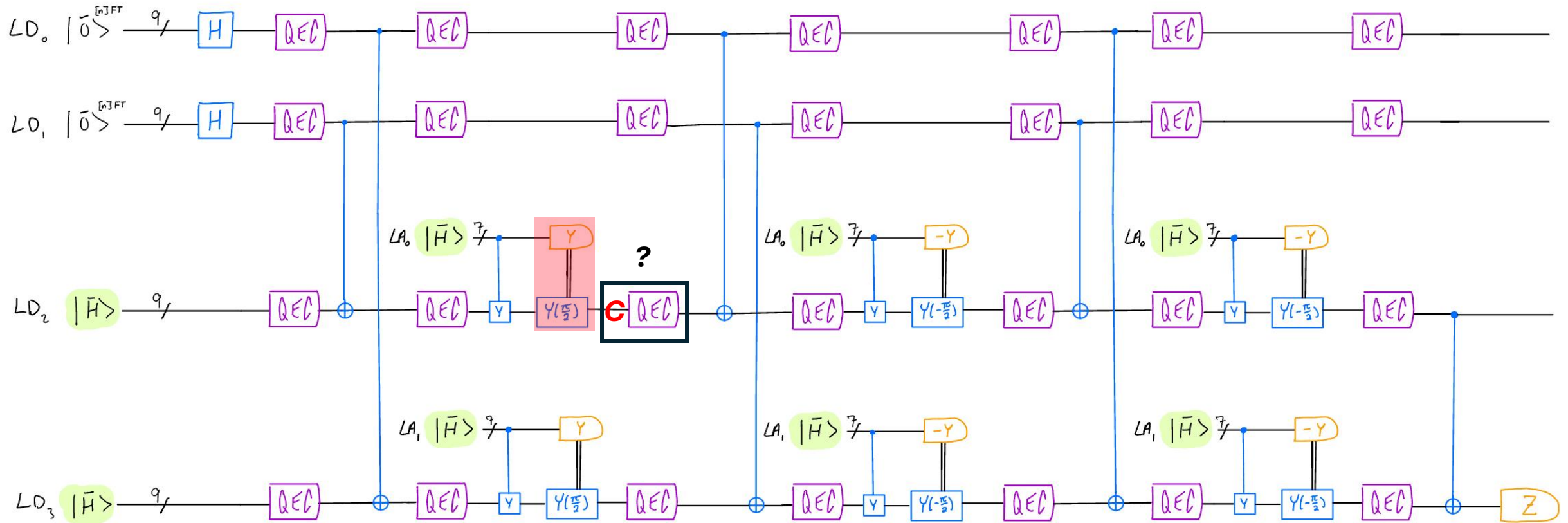
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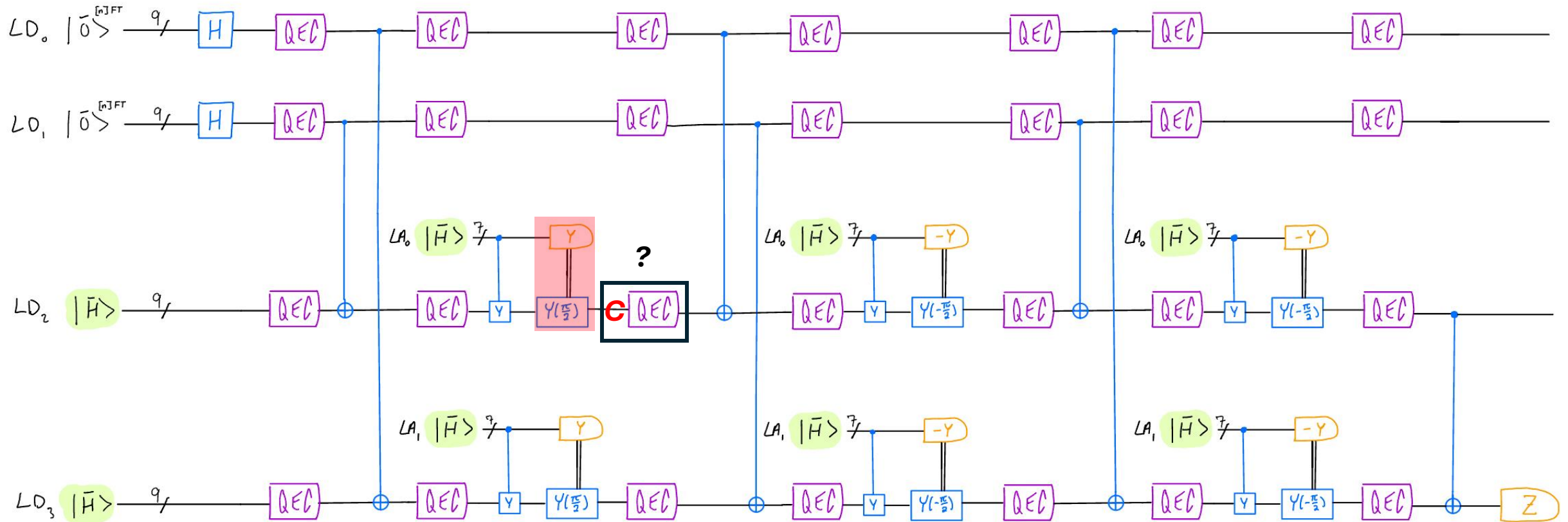
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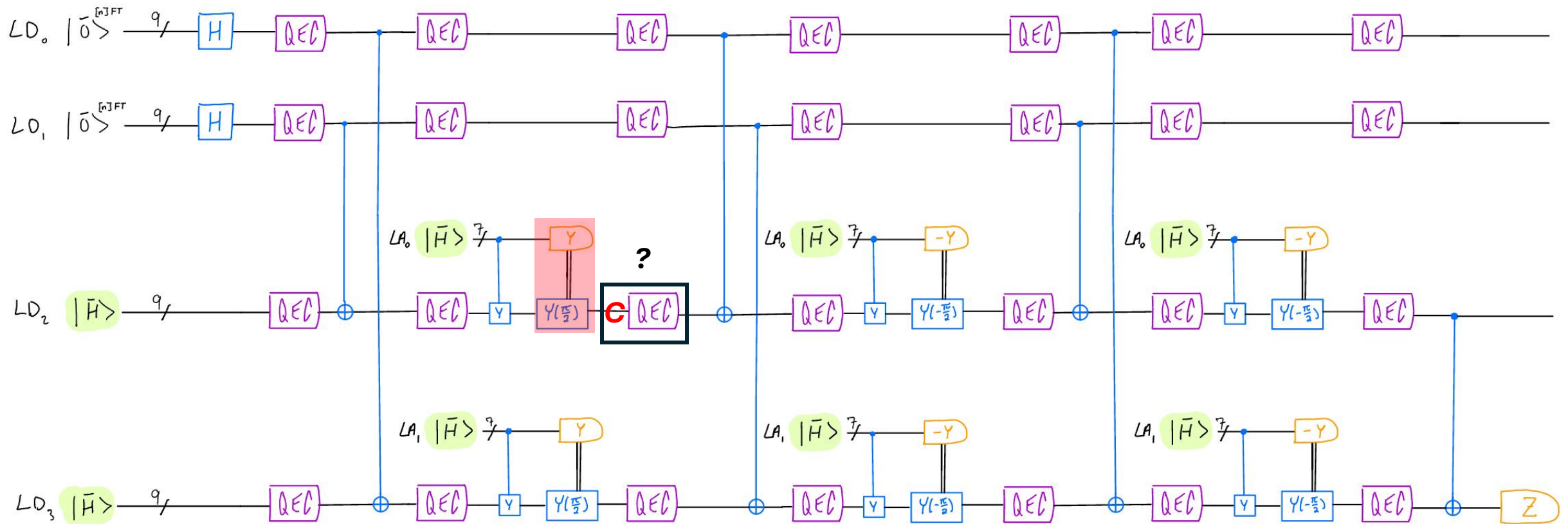


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could we somehow modify the injection circuit so that it only contains a Pauli correction?

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can we use the algorithmic fault tolerance discussed by Luis in journal club?